Multidimensional $D^2$ phase dispersion statistic

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AstroMHD Research Group Meeting
Monday 22nd August, 2016
Harmonics and periods

Spectral analysis
Thinking in terms of harmonics
- Power spectral density $\rightarrow$ periodogram
- Unevenly sampled data: Lomb-Scargle periodogram

PDM methods
Thinking in terms of periods
- By default support uneven sampling
- Generalizable to multiple dimensions

$$SS(P) = \sum_{i=1}^{n} (y_i(P) - \hat{y}_i(P))^2$$ (1)

Lafler & Kinman: $\hat{y}_i(P) = y_{i-1}(P)$, Renson: weighted version
Stellingwerf: $\hat{y}_i(P) = \bar{y}_i(P)$, where $\bar{y}_i(P)$ is a bin mean.
Examples of phase diagram

Correct period

Wrong period
From periods to cycles

$D^2$ method

Thinking in terms of cycles

$$D^2(P, t_{coh}) = rac{\sum_{i,j>i} g(t_i, t_j, P, t_{coh}) \|f(t_i) - f(t_j)\|^2}{2\sigma^2 \sum_{i,j>i} g(t_i, t_j, P, t_{coh})}, \quad (2)$$

where $g = g_1(t_i, t_j, P)g_2(t_i, t_j, t_{coh})$

- Essentially a generalized PDM method
- In addition to selection function $g_1$ in phase, introduces a selection function $g_2$ in time lag

We use:

$g_1 = 2 \cos(2\pi \nu (t_j - t_i)) + 1$

$g_2 = \exp(-\ln 2((t_j - t_i)/t_{coh})^2)$
Significance estimation

- Null hypothesis: Gaussian white noise
- Stellingwerf showed that the spectral line has then $F$ distribution.
- In many cases analytical form is hard or impossible to derive $\rightarrow$ use permutation test
- Is white noise correct null hypothesis?
- Also in red noise (or Brownian noise) patterns of periodicity can occur.
- Red noise is a special case of AR(1) process $X_t = c + \varphi X_{t-1} + \epsilon_t$ with $\varphi > 0$
- How to use red noise as null hypothesis?
- What if we could resample from the same process instead?
Example of red noise spectrum
Test cases

Rotating particle

Oscillations in a box

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Patterns

Periodic signal

(a) Multidimensional $D^2$ phase dispersion statistic

(b) Multidimensional $D^2$ phase dispersion statistic
Patterns

Cyclic signal

(a) Multidimensional $D^2$ phase dispersion statistic
(b) Multidimensional $D^2$ phase dispersion statistic

Value

Time

Value

Time

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Patterns

Temporary signal

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## Cycles in PENCIL-Millennium data

### Mean cycle length estimates

<table>
<thead>
<tr>
<th>Cycle no</th>
<th>$B_r$</th>
<th>$B_\theta$</th>
<th>$B_\phi$</th>
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<tbody>
<tr>
<td></td>
<td>N</td>
<td>S</td>
<td>N</td>
</tr>
<tr>
<td>I</td>
<td>0.47</td>
<td>0.47</td>
<td>0.48</td>
</tr>
<tr>
<td>II</td>
<td>5.12</td>
<td>4.98</td>
<td>5.13</td>
</tr>
<tr>
<td>III</td>
<td>49.2</td>
<td>43.0</td>
<td>46.2</td>
</tr>
<tr>
<td>IV</td>
<td>108.4</td>
<td>105.1</td>
<td>108.0</td>
</tr>
</tbody>
</table>

Notes: the numbers in *italic* represent cycles appearing only in the bottom of the convection zone, otherwise the cycle exists in the full hemisphere.
Cycles in PENCIL-Millennium data

5 year cycle

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Cycles in PENCIL-Millennium data

100 year cycle

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Cycles in PENCIL-Millennium data

0.5 year cycle
Cycles in PENCIL-Millennium data

50 year cycle

$\bar{B}_r$ (north)  $\bar{B}_r$ (south)

$\bar{B}_\theta$ (north)  $\bar{B}_\theta$ (south)

$\bar{B}_\phi$ (north)  $\bar{B}_\phi$ (south)