Impact of SAT-Based Preprocessing on Core-Guided MaxSAT Solving[1]

Jeremias Berg and Matti Järvisalo

HIIT, Dept. Computer Science
University of Helsinki
Finland

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Our Contributions

- Core guided MaxSAT solvers use a SAT-solver as a subroutine.
  - Understanding what factors affect number of calls important for developing efficient solvers.
- *Preprocessing:* essential in SAT-solving.
  - Motivates development of preprocessing for MaxSAT.
- We analyze the effect of preprocessing on the number of SAT calls required by core guided MaxSAT solvers.

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Our Results:

- Preprocessing has *no effect* on the best case number of iterations.
- Preprocessing can improve the worst case number of iterations.

Outline

- Background / Motivation
- Satisfiability (SAT) and Maximum Satisfiability (MaxSAT)
- Preprocessing SAT
- Preprocessing MaxSAT
- Research question
- Abstractions of MaxSAT solvers
- Results
- Proof intuition
- Summary

Background / Motivation

- Maximum Satisfiability
  - The optimization counterpart of the Satisfiability problem
- Exact MaxSAT solving is an active area of research.
- Solvers have improved significantly over the last years.
- Applications in: inconsistency analysis, diagnosis, design debugging, and fault localization, AI, combinatorics, data analysis, bioinformatics, ...

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MaxSAT for real world problems

- Most MaxSAT solvers for industrial problems use of a SAT solver as a subroutine.
  - Unsat Core extraction
- Potential speedups: faster or fewer SAT solver calls.
- *Preprocessing:* an essential part of SAT-solving
- Currently not as well understood for MaxSAT.

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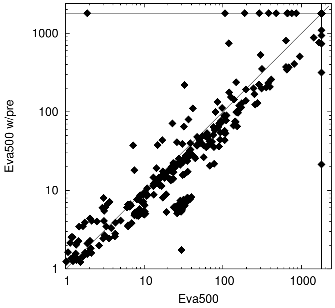
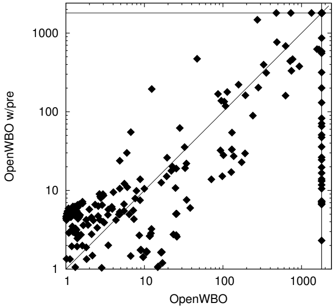
Effect of Preprocessing





- Solvers: Eva500 and Open-Wbo

  [Narodytska and Bacchus, 2014; Martins, Joshi, Manquinho, and Lynce, 2014a]

- Preprocessing + Solving time on Y-axis.

Our Contributions

- Formal Analysis of the effect of preprocessing on MaxSAT solving.

Results

- Neither preprocessing (nor core-minimization) has *any effect* on the best case performance.
- Preprocessing (and core minimization) can improve the worst case performance of both solvers,

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- Formal Analysis of the effect of preprocessing on MaxSAT solving.
  - The effect of preprocessing on the number of SAT-solver calls required by two abstractions of MaxSAT solvers
  - As a byproduct, similar analysis on core minimization.

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- Satisfiability (SAT): Given a CNF-formula F, decide if its satisfiable

$$F = \quad (y \vee \neg z) \wedge (x \vee z) \wedge (\neg z) \wedge (y \vee z) \wedge (\neg x) \wedge (\neg y \vee \neg z) \wedge (z \vee y)$$

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$$F = \{\{y, \neg z\}, \{x, z\}, \{\neg z\}, \{y, z\}, \{\neg x\}, \{\neg y, \neg z\}, \{z, y\}\}$$

Maximum Satisfiability

- (Partial) Maximum Satisfiability (MaxSAT):
  - Two CNF-Formulas: F_h, (hard clauses) and F_s (soft clauses)

$$F_h = \{(x \vee z), (\neg z), (y \vee z)\}$$

$$F_s = \{(\neg x), (\neg y \vee \neg z), (z \vee y), (\neg z \vee y)\}$$

Maximum Satisfiability

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  - Two CNF-Formulas: F_h, (hard clauses) and F_s (soft clauses)
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$$F_h = \{(x \lor z), (\neg z), (y \lor z)\}$$

$$F_s = \{((\neg x), 1), ((\neg y \lor \neg z), 3), ((z \lor y), 7), ((\neg z \lor y), 3)\}$$

Maximum Satisfiability

- (Weighted Partial) Maximum Satisfiability (MaxSAT):
  - Two CNF-Formulas: F_h, (hard clauses) and F_s (soft clauses)
  - Find a truth assignment satisfying F_h and a maximum sum of weights of clauses in F_s.
- A (minimal) unsatisfiable core (MUS) C: a (subset minimal) $C \subseteq F_s$ s.t. $C \wedge F_h$ is unsatisfiable.
  - $mus(F)$: the set of all MUSes of F.

$$F_h = \{(\mathbf{x} \vee \mathbf{z}), (\neg \mathbf{z}), (y \vee z)\}$$

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Preprocessing SAT

Preprocessing pipeline:

1. Apply (fast) satisfiability preserving simplifications to F to obtain $pre(F)$.
2. Solve $pre(F)$.
3. Reconstruct solution to F (if needed).

- Subsumption Elimination (SE)
  - If there exists clauses $C, D \in F$ s.t. $C \subseteq D$, remove D.
- Bounded Variable Elimination (BVE) [Eén and Biere, 2005]
- Self Subsuming Resolution (SSR)
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SAT-based preprocessing

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For the rest of this talk: **SAT-based preprocessing techniques**

Preprocessing MaxSAT

- SAT-based preprocessing techniques have been lifted to MaxSAT.
  [Belov, Morgado, and Marques-Silva, 2013]
  - Requires fresh "label[2]" variables on soft clauses.

[2] Assumption, Reification

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Preprocessing a MaxSAT instance (F_h, F_s)

1. Run SAT-preprocessor on $F_h \cup F_s^a$ where $F_s^a = \{C \lor l_C \mid C \in F_s\}$
   - Do not resolve on l_C variables.
   - Output $pre(F)_h$
2. $pre(F_s) = \{(\neg l_C) \mid C \in F_s\}$
3. Solve $(pre(F)_h, pre(F)_s)$

[2] Assumption, Reification

Effect of Preprocessing

For a fixed MaxSAT algorithm $\mathcal{A}$, consider:

1. $\mathcal{A}$
2. $\mathcal{A}_{\text{pre}}$: $\mathcal{A}$ + SAT based preprocessing.
3. $\mathcal{A}^{mus}$: $\mathcal{A}$ using a SAT solver that is guaranteed to return a MUS when invoked on an unsatisfiable formula
4. $\mathcal{A}^{mus}_{\text{pre}}$: $\mathcal{A}^{mus}$ + SAT based preprocessing.

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Our Research Question

How does SAT based preprocessing affect the number of SAT-solver calls required by these variants for $\mathcal{A} \in \{\text{CG}, \text{HS}\}$?

The abstract MaxSAT solvers we analyze in this work.

Bacchus and Narodytska [2014]

CG:
$F_w^1 \leftarrow F_h \cup F_s$
for *i=1...* **do**
  $(\textit{result}, \kappa, \tau) \leftarrow \text{SATSOLVE}(F_w^i)$
  if *result="satisfiable"* **then**
    return τ // optimal solution
  else
    // κ = unsat core
    $F_w^i = (F_w^i \setminus \kappa)$
    $F_w^{i+1} \leftarrow \text{RELAX}(F_w^i, \kappa)$
  end
end

- Core guided solver.
- Iteratively extracts cores from the instances and relaxes them.
- Refines a lower bound on the optimal cost
- Instantiated as: Fu-Malik, WPM1, MSU3, ...

The abstract MaxSAT solvers we analyze in this work.

Davies and Bacchus [2013]; Saikko et al. [2016]

- Implicit hitting set approach to MaxSAT.
- Iteratively extracts cores from the instances and computes hitting sets over the set of found cores.
- Instantiated as: MaxHS, LMHS

HS:

$\mathcal{K} \leftarrow \emptyset$ // set of found unsat cores of F
$F_w \leftarrow (F_h \cup F_s)$
while *true* **do**
  $H \leftarrow$ MINCOSTHITTINGSET($\mathcal{K}$)
  $F_w \leftarrow F_h \cup (F_s \setminus H)$
  (*result*, κ, τ) $\leftarrow$ SATSOLVE(F_w)
  if *result="satisfiable"* **then**
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Example CG: WPM1

Manquinho et al. [2009]; Ansótegui et al. [2009]; Fu and Malik [2006]

$$F_h = \{(x_1 \vee x_2 \vee x_3), (x_3 \vee x_4 \vee x_5), (x_5 \vee x_6 \vee x_7)\}$$

$$F_s = \{(\neg x_1), (\neg x_2), (\neg x_3), (\neg x_4), (\neg x_5), (\neg x_6), (\neg x_7)\}$$

Input

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SATISFIABLE
2 original soft clauses unsatisfied

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Example HS: Davies and Bacchus [2013]; Saikko et al. [2016]

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Second Hitting Set $\{(\neg x_5)\}$

Set of Cores:

$\{(\neg x_3), (\neg x_4), (\neg x_5)\}$

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Example HS: Davies and Bacchus [2013]; Saikko et al. [2016]

$$F_h = \{(x_1 \lor x_2 \lor x_3), (x_3 \lor x_4 \lor x_5), (x_5 \lor x_6 \lor x_7)\}$$

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Third Core $\{(\neg x_1), (\neg x_2), (\neg x_3)\}$

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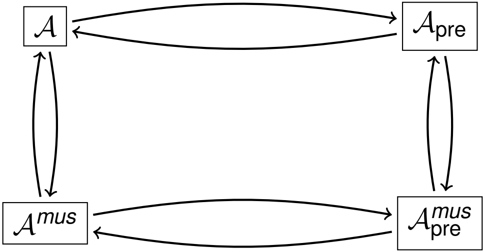
SATISFIABLE
2 original soft clauses unsatisfied

Our Results, Best-Case Performance

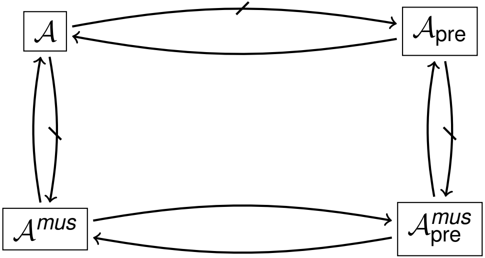
For $\mathcal{A} \in \{\text{CG}, \text{HS}\}$:



$X \rightarrow Y$ iff MINITERATIONS$(X) \leq$ MINITERATIONS(Y)

Our Results, Worst-Case Performance

For $\mathcal{A} \in \{\text{CG}, \text{HS}\}$:



$X \rightarrow Y$ iff $\text{MaxIterations}(X) \leq \text{MaxIterations}(Y)$

$X \nrightarrow Y$ indicates that $X \rightarrow Y$ does not hold.

Proof intuition

Belov et al. [2013]: Preprocess a MaxSAT instance F using SAT-based preprocessing to obtain $pre(F)$. Then

$$mus(F) = mus(pre(F)). \tag{1}$$

We show:

- The shortest executions of all solver variants require the SAT solver only extracting MUSes. Best Case results
- There are instances on which extracting non minimal cores forces both algorithms to iterate unnecessary many times. Worst case results

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Note: Results hold for any preprocessing techniques satisfying Eq. 1.

Summary

- Understanding what factors affect the number of SAT solver calls is important for developing more efficient MaxSAT solvers.
- In this work: Effect of SAT-based preprocessing on the number of iterations
  - No effect on the best case.
  - Can improve the worst case.
- Further work:
  - Similar analysis for other MaxSAT algorithms (MaxRES, OLL, . . .).
  - Effect of preprocessing on individual SAT-solver calls.
  - Development of other MaxSAT preprocessing techniques.

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