Learning Moore Machines from Input-Output Traces

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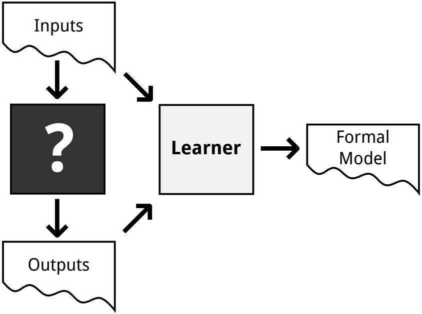
Motivation: learning models from black boxes



Many applications:

- Verify that a black-box component is safe to use
- Dynamic malware analysis
- ...

Learning FSMs from input-output traces

IO-traces

$aa \mapsto 020$

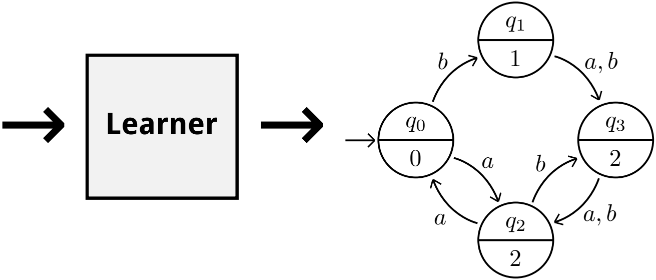
$baa \mapsto 0122$

$bba \mapsto 0122$

$abaa \mapsto 02220$

$abba \mapsto 02220$

Learned FSM



Outline

1. Background
2. Formal problem definition
3. Related work
4. Identification in the limit
5. Our learning algorithms
6. Results
7. Summary & future work

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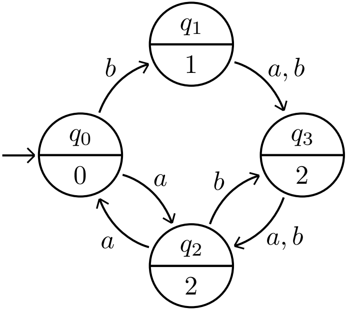
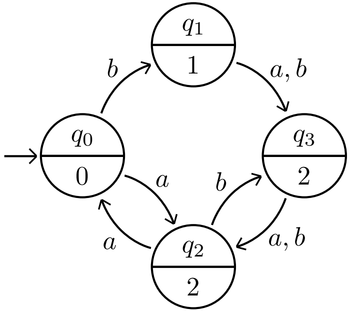
Moore machines



$(I, O, Q, q_0, \delta, \lambda)$

- input alphabet, $I = \{a, b\}$
- output alphabet, $O = \{0, 1, 2\}$
- set of states, $Q = \{q_0, q_1, q_2, q_3\}$
- initial state, q_0
- transition function, $\delta : Q \times I \to Q$
- output function, $\lambda : Q \to O$

Moore machines



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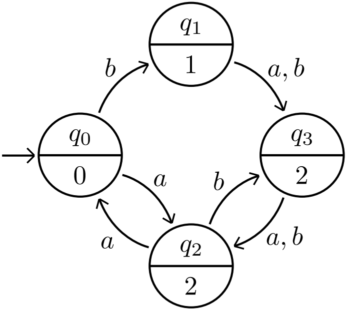
By definition, our machines are **deterministic** and **complete**.

Input-output traces



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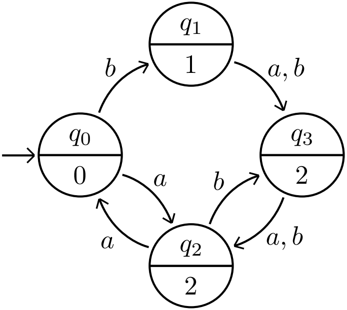
Moore machine

Some I/O traces generated by the machine

Consistency



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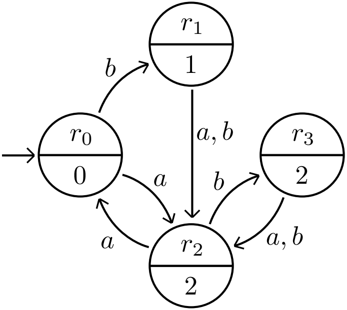
This machine is consistent with this set of traces.

Consistency



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A first attempt at problem definition

Given ...

- Input alphabet, I
- Output alphabet, O
- Set of IO-traces, S (the **training set**)

... find a Moore machine M such that:

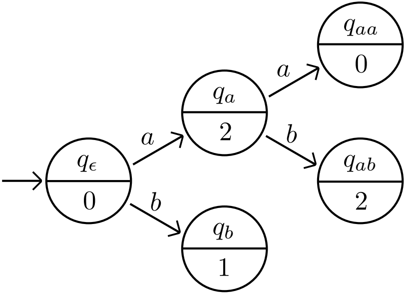
- M is deterministic
- M is complete
- M is consistent with S

A trivial solution

$b \mapsto 01$

$aa \mapsto 020$

$ab \mapsto 022$



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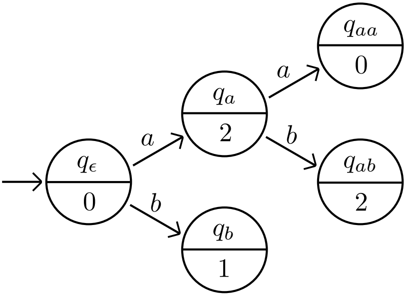

This is called the **prefix-tree machine**.

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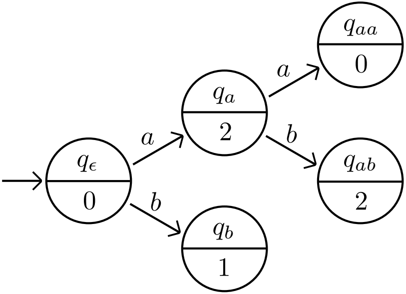

This is called the **prefix-tree machine**.
Not quite a solution: machine incomplete ...

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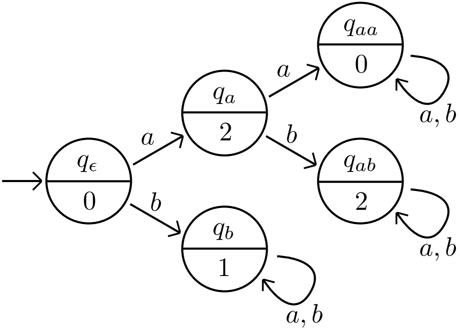

... but easily completed with self-loops.

Problems with the trivial solution

(1) **Poor generalization**, due to trivial completion with self-loops

- The machine may be consistent with the **training** set ...
- ... but how **accurate** is it on a **test** set?

Problems with the trivial solution

(1) **Poor generalization**, due to trivial completion with self-loops

- The machine may be consistent with the **training** set ...
- ... but how **accurate** is it on a **test** set?

(2) Large number of states in the learned machine

- The prefix-tree machine does not merge states at all.

Revised problem definition

The LMoMIO problem (Learning Moore Machines Input-Output Traces):

Given ...

- Input alphabet, I
- Output alphabet, O
- Set of IO-traces, S (the training set)

... find a Moore machine M such that:

- M is deterministic
- M is complete
- M is consistent with S

... and also:

- M generalizes well (good accuracy on a-priori unknown test sets)
- M is small (few states)
- M is found quickly (good learning algorithm complexity)

How to measure "accuracy"?

We define three metrics: **Strong**, **Medium**, **Weak**

test trace	machine output	strong acc.	medium acc.	weak acc.
$abc \mapsto 1234$	1234	1	1	1
$abc \mapsto 1234$	4321	0	0	0
$abc \mapsto 1234$	1212	0	$\frac{1}{2}$	$\frac{1}{2}$
$abc \mapsto 1234$	3434	0	0	$\frac{1}{2}$
$abc \mapsto 1234$	1324	0	$\frac{1}{4}$	$\frac{1}{2}$

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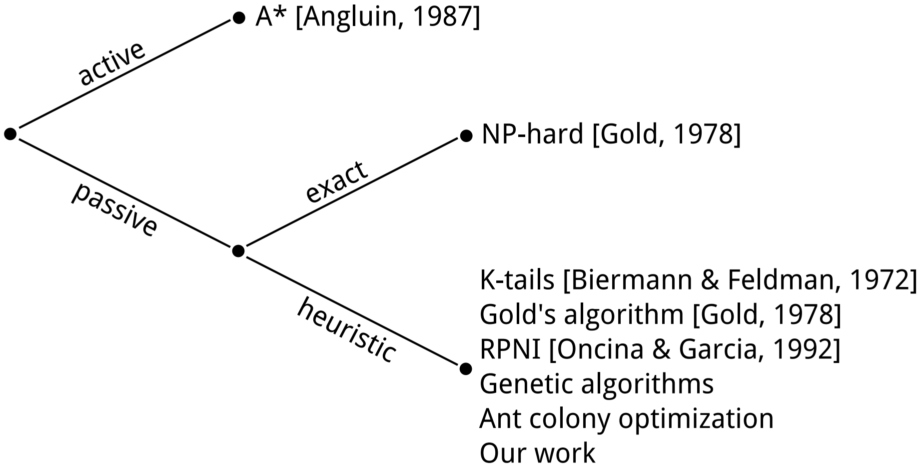
Related work



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Identification in the limit

Concept introduced in [Gold, 1967], in the context of formal language learning

- Learning is seen as an infinite process
- Training set keeps growing: $S_0 \subseteq S_1 \subseteq S_2 \subseteq \cdots$
- Every input word is guaranteed to eventually appear in the training set
- For each S_i, the learner outputs machine M_i
- Identification in the limit $:=$ learner outputs the right machine after some i

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A good passive learning algorithm must identify in the limit.

Characteristic samples

To prove identification in the limit, we use the notion of the **Characteristic Sample** [C. de la Higuera, 2010]:

- Concept existing for DFAs (deterministic finite automata) – we adapt it to Moore machines
- Intuition: set of IO-traces that "covers" the machine (covers all states, all transitions)
- For a minimal Moore machine $M = (I, O, Q, q_0, \delta, \lambda)$, there exists a CS of total length $O(|Q|^4|I|)$

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Charateristic Sample Requirement (CSR):

- A learning algorithm satisfies CSR if it satisfies the following:

  If the training set S is a characteristic sample of a minimal machine M, then the algorithm learns from S a machine isomorphic to M.

- CSR can be shown to imply identification in the limit

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Three learning algorithms

- PTAP - Prefix Tree Acceptor Product
- PRPNI - Product RPNI
- MooreMI - Moore Machine Inference

PTAP - Prefix Tree Acceptor Product

This is the trivial solution we discussed earlier:

$b \mapsto 01$

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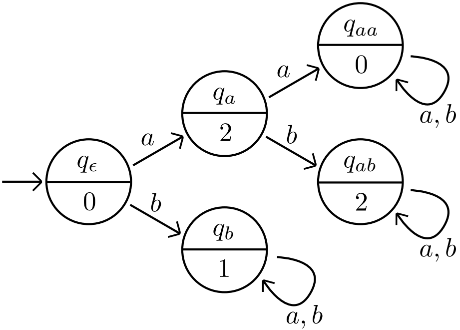

Drawbacks:

- Large number of states in learned machine
- Poor generalization / accuracy

PRPNI - Product RPNI

Observations:

- A DFA is a special case of a Moore machine with binary output (accept/reject)
- A Moore machine can be encoded as a product of $\lceil \log_2 |O| \rceil$ DFAs

Based on these observations, PRPNI works as follows:

- Uses the RPNI algorithm [J. Oncina and P. Garcia, 1992], which learns DFAs
- Learns several DFAs that encode the learned Moore machine
- Computes product of the learned DFAs and completes it

Drawbacks:

- DFAs are learned separately, therefore do not have same state-transition structure $\implies$ state explosion during product computation

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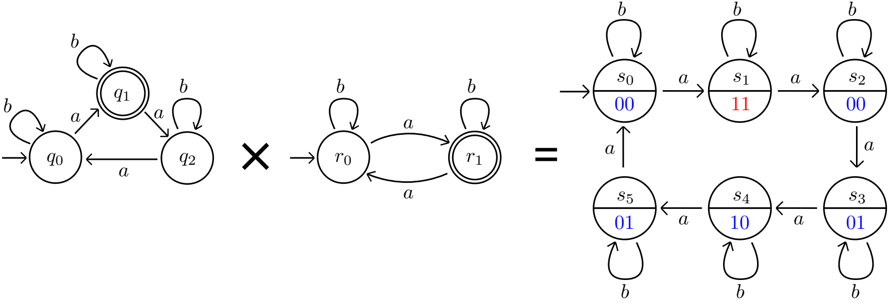
Drawbacks:

- DFAs are learned separately, therefore do not have same state-transition structure $\implies$ state explosion during product computation
- Invalid output codes

Invalid output codes

Output alphabet: $O = \{0, 1, 2\}$

Binary encoding of O: $f = \{0 \mapsto 00, 1 \mapsto 01, 2 \mapsto 10\}$



Invalid output code: 11 does not correspond to any output symbol

MooreMI - Moore Machine Inference

- Modified RPNI, tailored to Moore machine learning
- Like PRPNI, learns several DFAs that encode the learned Moore machine
- Unlike PRPNI, learned DFAs maintain same state-transition structure
- Therefore, no state explosion during product computation
- No invalid output codes either

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Results

Theorem 1

All three algorithms return Moore machines consistent with the IO-traces received as input.

Theorem 2

The MooreMI algorithm satisfies the characteristic sample requirement and identifies in the limit.

Experimental evaluation result:

MooreMI is better not just in theory, but also in practice

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Summary

- Learning deterministic, complete Moore machines from input-output traces
- Characteristic sample for Moore machines
- Three algorithms to solve the problem
- MooreMI algorithm identifies in the limit

Future work

- Extend to Mealy machines
- Learning symbolic machines
- Learning from traces and formal requirements (e.g. LTL formulas)
- Industrial case studies

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- Learning symbolic machines
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Thank you! Questions?

References

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