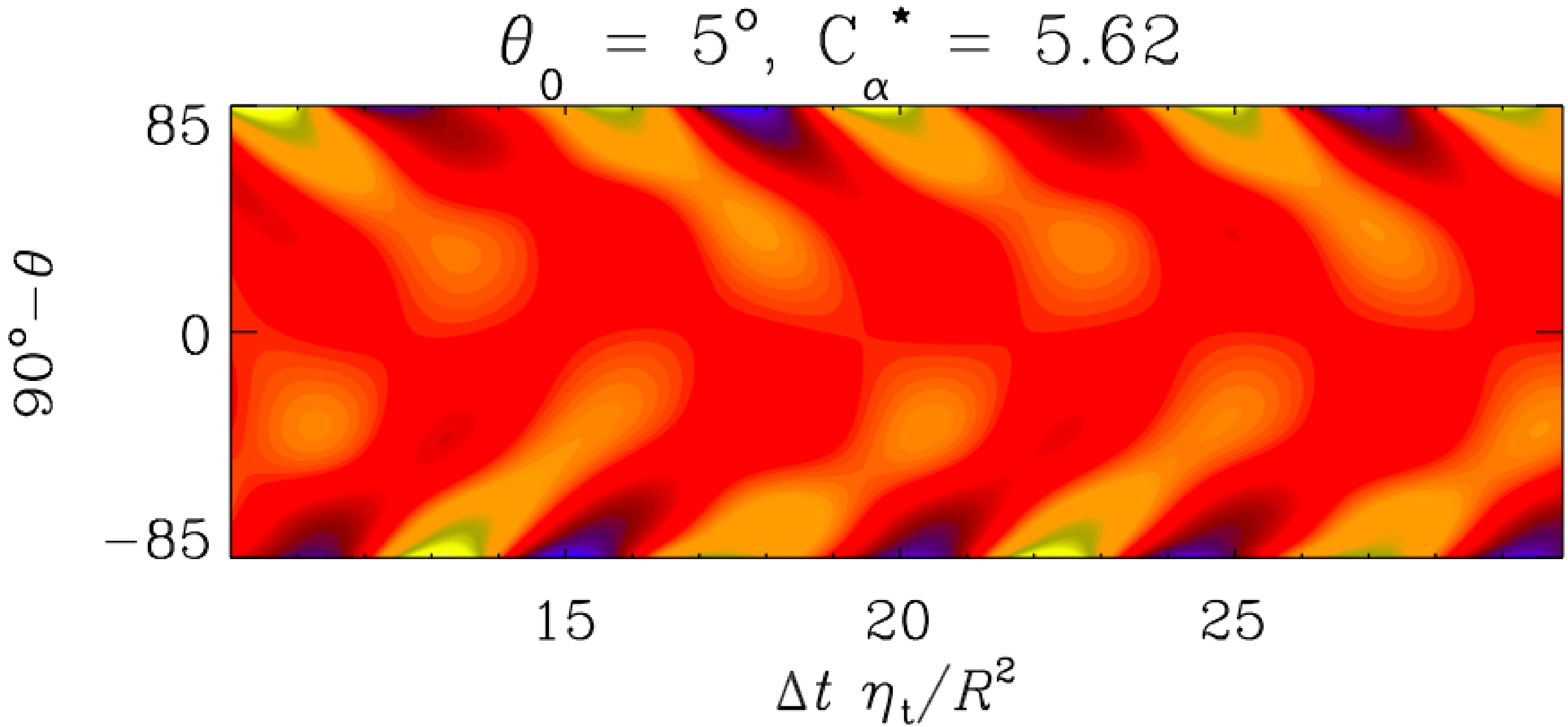
Oscillatory α² Dynamos



Background

Start with meanfield dynamo equation:

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta \mu_0 \overline{\boldsymbol{J}}\right)$$

Background

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta \mu_0 \overline{\boldsymbol{J}} \right)$$

Meanflow from
angular velocity

$$\overline{\boldsymbol{U}} \;=\; \hat{\phi} \varpi \Omega$$

Background

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta \mu_0 \overline{\boldsymbol{J}} \right)$$

Meanflow from
angular velocity

$$\overline{\boldsymbol{U}} = \hat{\phi} \varpi \Omega$$

Mean magnetic field

Background

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta \mu_0 \overline{\boldsymbol{J}} \right)$$

Meanflow from angular velocity

$$\overline{\boldsymbol{U}} \;=\; \hat{\phi} \varpi \Omega$$

Mean EMF

$$\overline{\boldsymbol{\mathcal{E}}} = \alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \mu_0 \overline{\boldsymbol{J}}.$$

Mean magnetic field

Background

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta \mu_0 \overline{\boldsymbol{J}}\right)$$

Meanflow from angular velocity

$$\overline{\boldsymbol{U}} = \hat{\phi} \varpi \Omega$$

Mean EMF

$$\overline{\boldsymbol{\mathcal{E}}} = \alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \mu_0 \overline{\boldsymbol{J}}.$$

Mean magnetic field

turbulent magnetic diffusivity

Background

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta\mu_0 \overline{\boldsymbol{J}}\right)$$

Meanflow from angular velocity

$$\overline{\boldsymbol{U}} = \hat{\phi}\varpi\Omega$$

Mean magnetic field

Mean EMF

$$\overline{\boldsymbol{\mathcal{E}}} = \alpha\overline{\boldsymbol{B}} - \eta_{\rm t}\mu_0\overline{\boldsymbol{J}}.$$

turbulent magnetic diffusivity

Current density

$$\boldsymbol{\nabla} \times \overline{\boldsymbol{B}}/\mu_0$$

Background

vacuum permeability

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta \mu_0 \overline{\boldsymbol{J}} \right)$$

Meanflow from angular velocity

$$\overline{\boldsymbol{U}} = \hat{\phi} \varpi \Omega$$

Mean EMF

$$\overline{\boldsymbol{\mathcal{E}}} = \alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \mu_0 \overline{\boldsymbol{J}}.$$

Current density

$$\boldsymbol{\nabla} \times \overline{\boldsymbol{B}} / \mu_0$$

Mean magnetic field

turbulent magnetic diffusivity

Background

non-turbulent magnetic diffusion

vacuum permeability

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta \mu_0 \overline{\boldsymbol{J}} \right)$$

Meanflow from angular velocity

$$\overline{\boldsymbol{U}} = \hat{\phi} \varpi \Omega$$

Mean EMF

$$\overline{\boldsymbol{\mathcal{E}}} = \alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \mu_0 \overline{\boldsymbol{J}}.$$

Current density

$$\boldsymbol{\nabla} \times \overline{\boldsymbol{B}} / \mu_0$$

Mean magnetic field

turbulent magnetic diffusivity

Background

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta\mu_0 \overline{\boldsymbol{J}}\right)$$

Pencil Code solves in terms of $\overline{\boldsymbol{A}}$:

$$\overline{\boldsymbol{B}} = \boldsymbol{\nabla} \times \overline{\boldsymbol{A}}$$

$$\frac{\partial \overline{\boldsymbol{A}}}{\partial t} = -\varpi \overline{A}_\phi \boldsymbol{\nabla}\Omega + \overline{\boldsymbol{\mathcal{E}}} - \eta\mu_0 \overline{\boldsymbol{J}}$$

α^2 Dynamos: vary θ and change boundary

SAA: regularity on θ_0

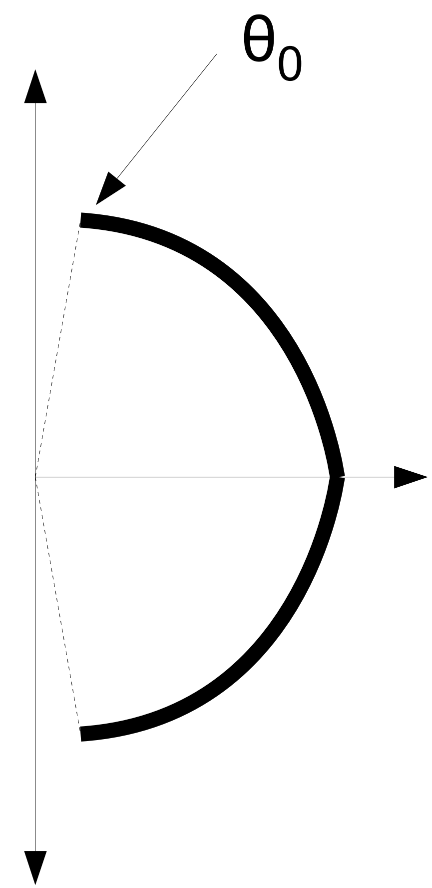
$$\partial_\theta \overline{A}_r = \overline{A}_\theta = \overline{A}_\phi = 0$$

ASA: perfect conductor on θ_0

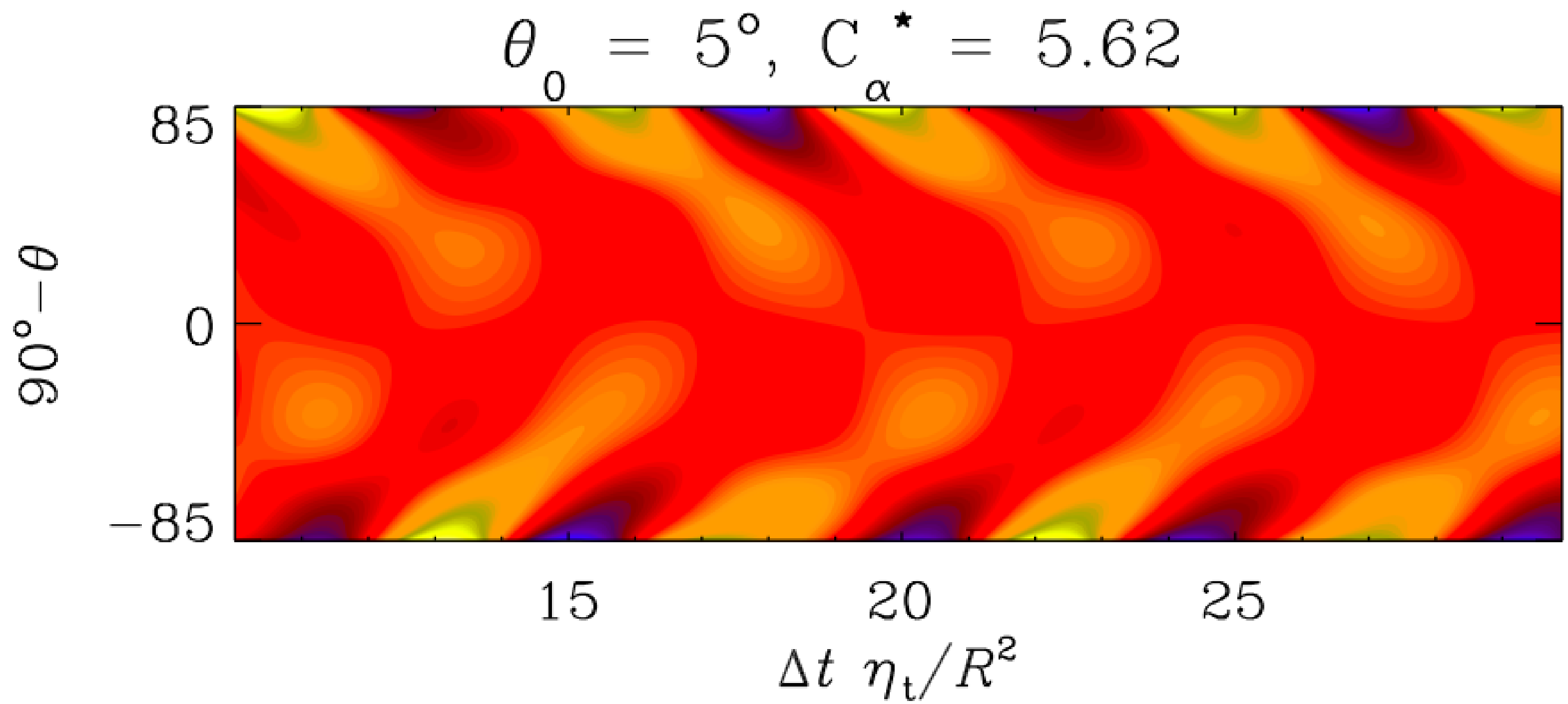
$$\overline{A}_r = \partial_\theta \overline{A}_\theta = \overline{A}_\phi = 0$$

SAS: normal field on θ_0

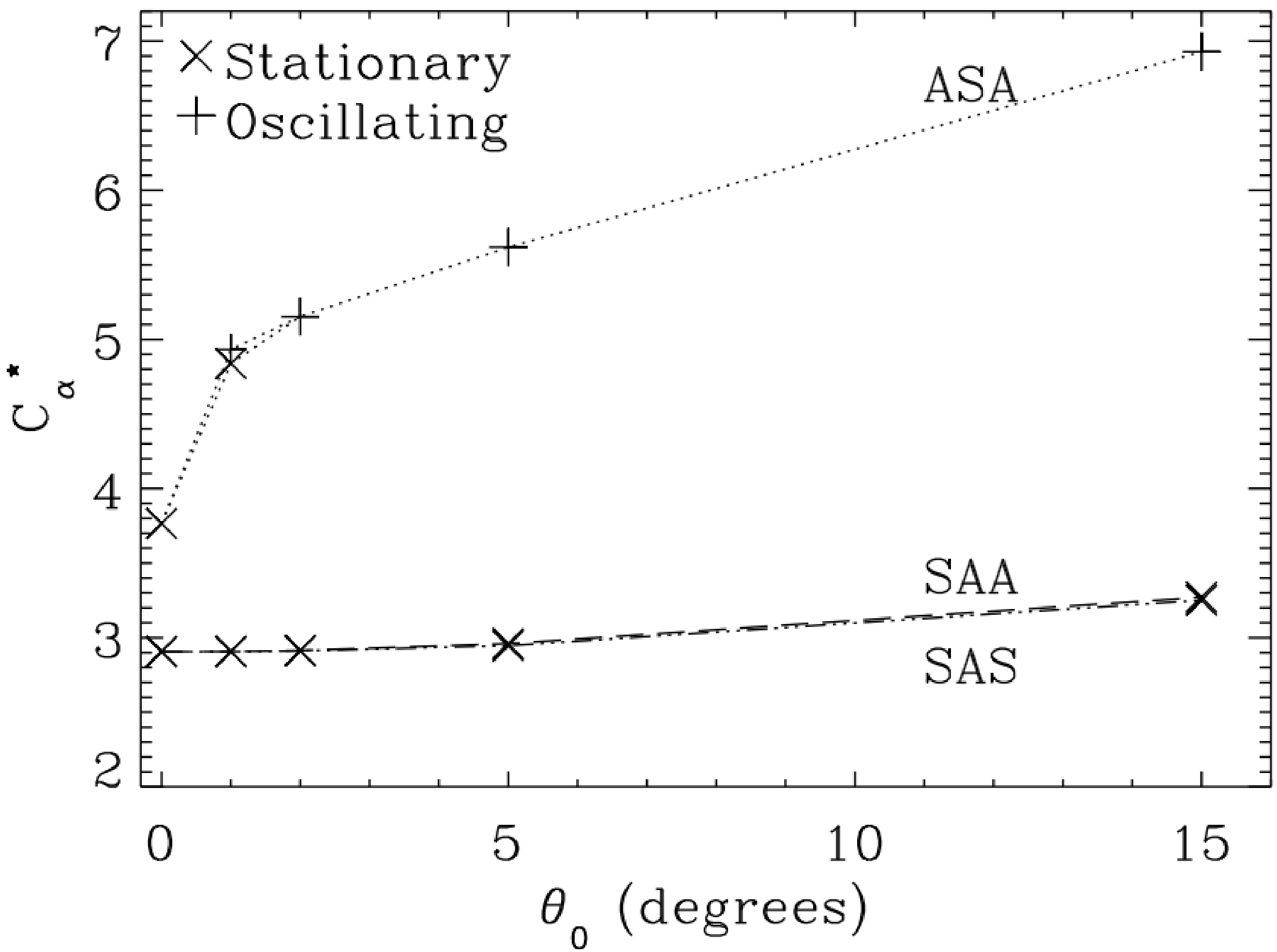
$$\partial_\theta \overline{A}_r = \overline{A}_\theta = \partial_\theta \overline{A}_\phi = 0$$



α^2 Dynamos: vary θ and change boundary

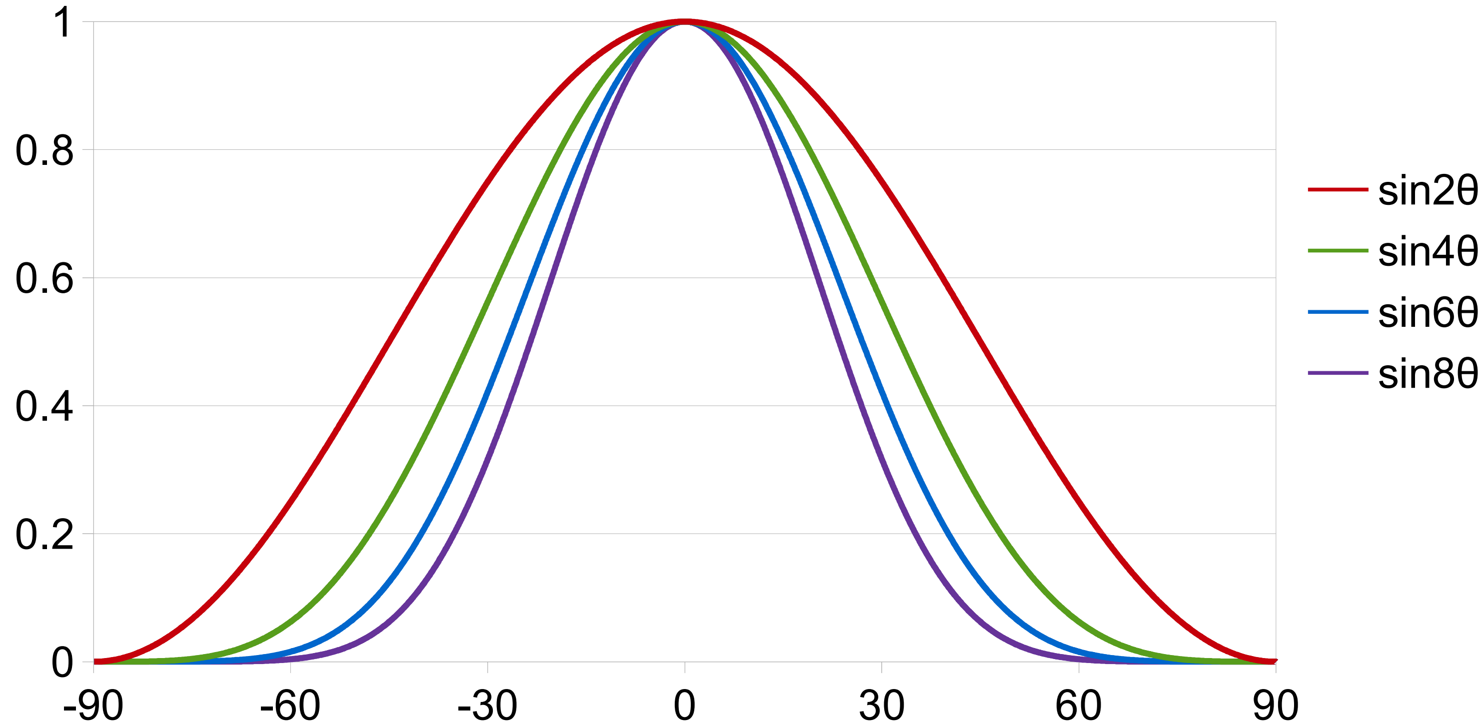


α^2 Dynamos: vary θ and change boundary

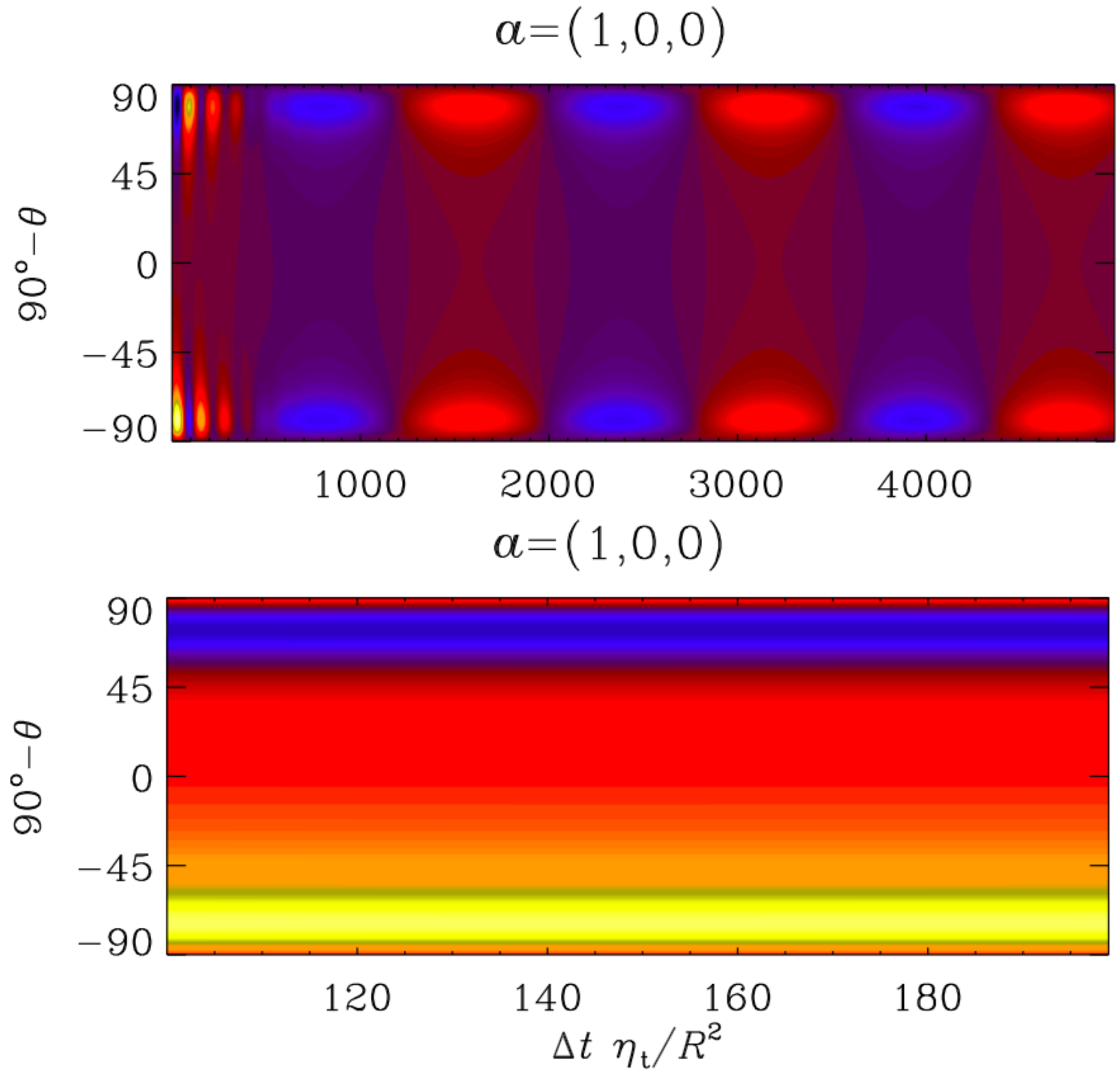


α^2 Dynamos: vary conductivity profile

$$\eta_t = \eta_{t0}\left(e_0 + e_2 \sin^2\theta + \ldots + e_n \sin^n\theta\right)$$



α^2 Dynamos: vary conductivity profile

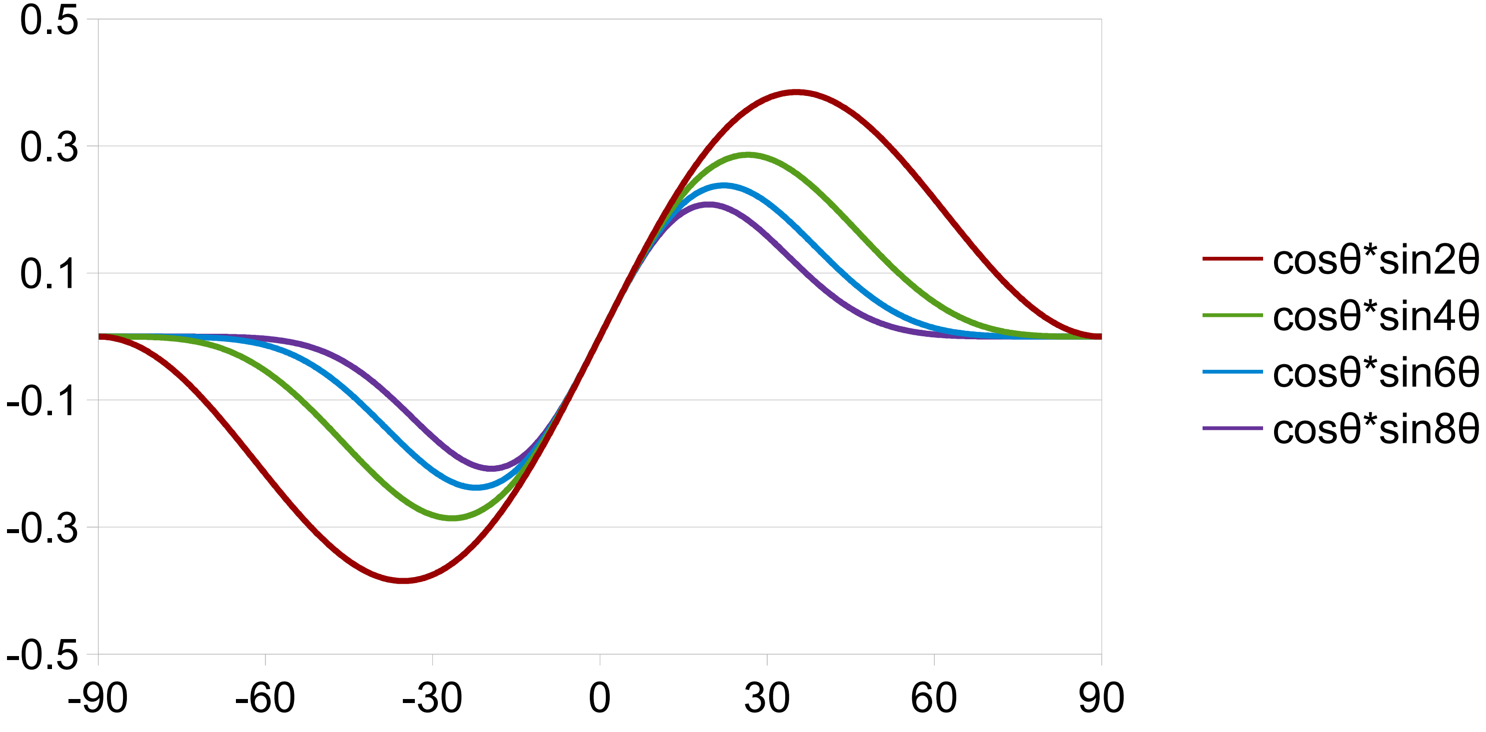


α^2 Dynamos: vary conductivity profile

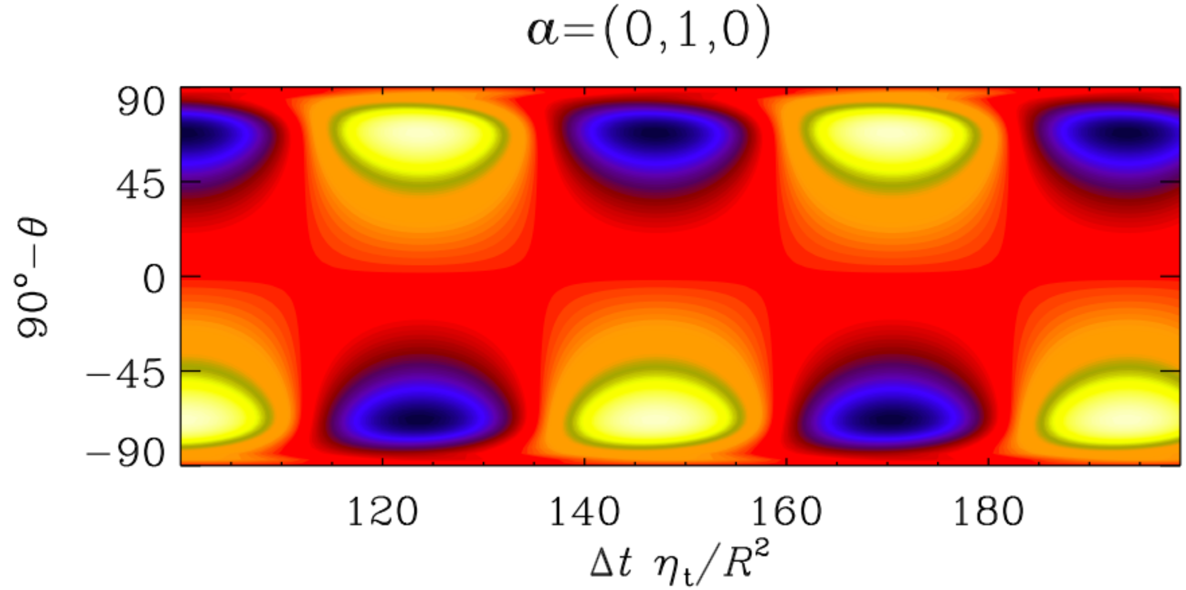
e_i	η/η_{t0}	$\boldsymbol{a}$ $(1,0,0)$ $C_\alpha^\star \mid \omega$
e_2	0.01	(0.236)
e_2	0.05	(0.558)
e_4	0.01	0.096 | 0.008
e_4	0.05	(0.326)
e_6	0.01	0.070 | 0.005
e_6	0.05	(0.265)
e_8	0.01	0.059 | 0.003
e_8	0.05	(0.238)

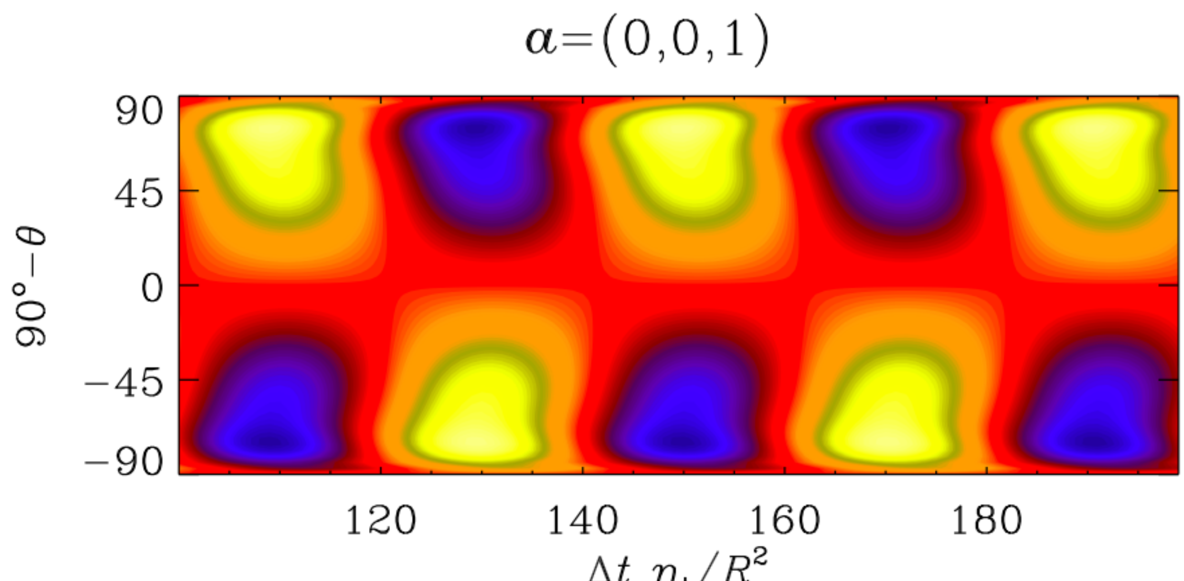
α^2 Dynamos: vary α profile

$$\alpha = \alpha_0 \cos\theta \left(a_0 + a_2 \sin^2\theta + \ldots + a_n \sin^n\theta\right)$$

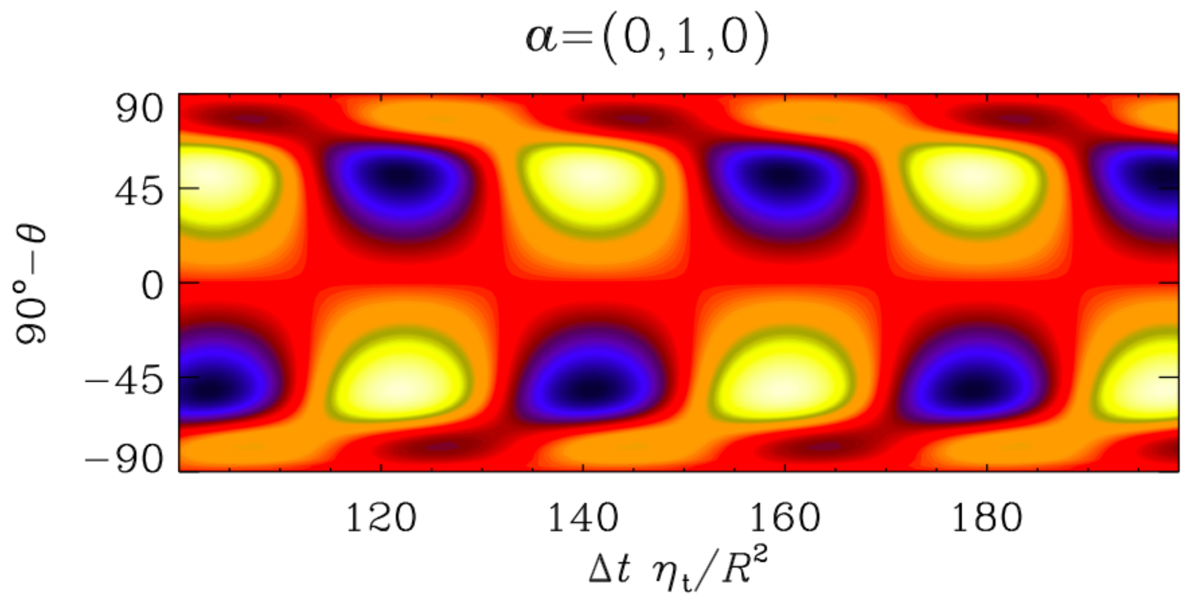


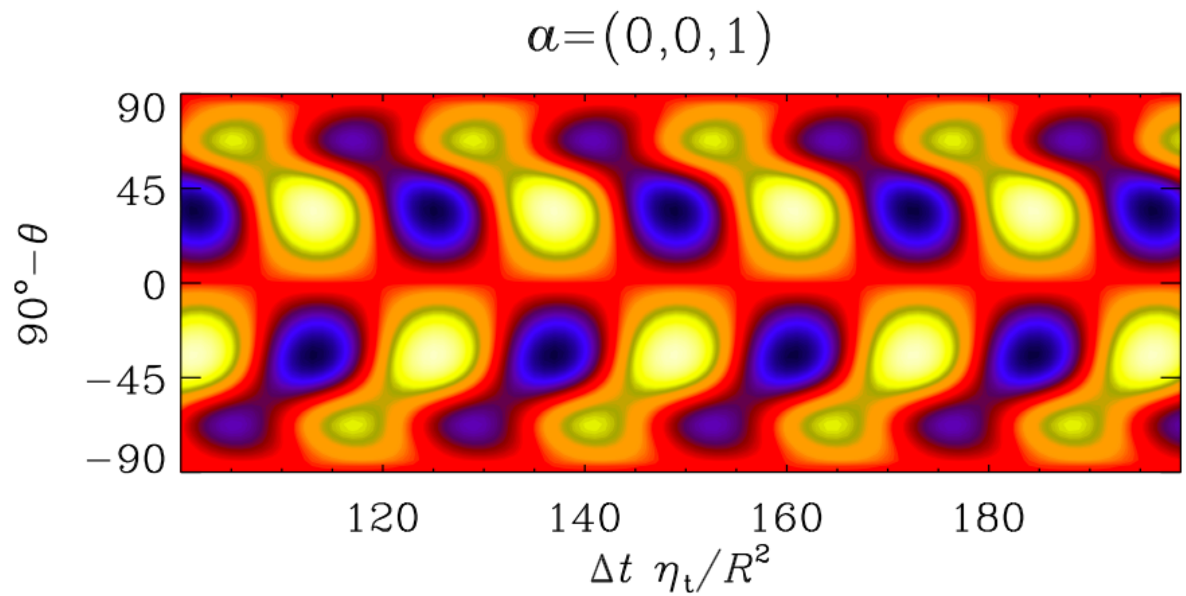
α^2 Dynamos: vary α profile





α^2 Dynamos: vary α profile





α^{2}Ω Dynamos: adding shear

$$\frac{\partial \overline{\boldsymbol{A}}}{\partial t} = -\varpi \overline{A}_\phi \boldsymbol{\nabla}\Omega + \overline{\boldsymbol{\mathcal{E}}} - \eta\mu_0 \overline{\boldsymbol{J}} - \mu^2 \overline{\boldsymbol{A}}$$

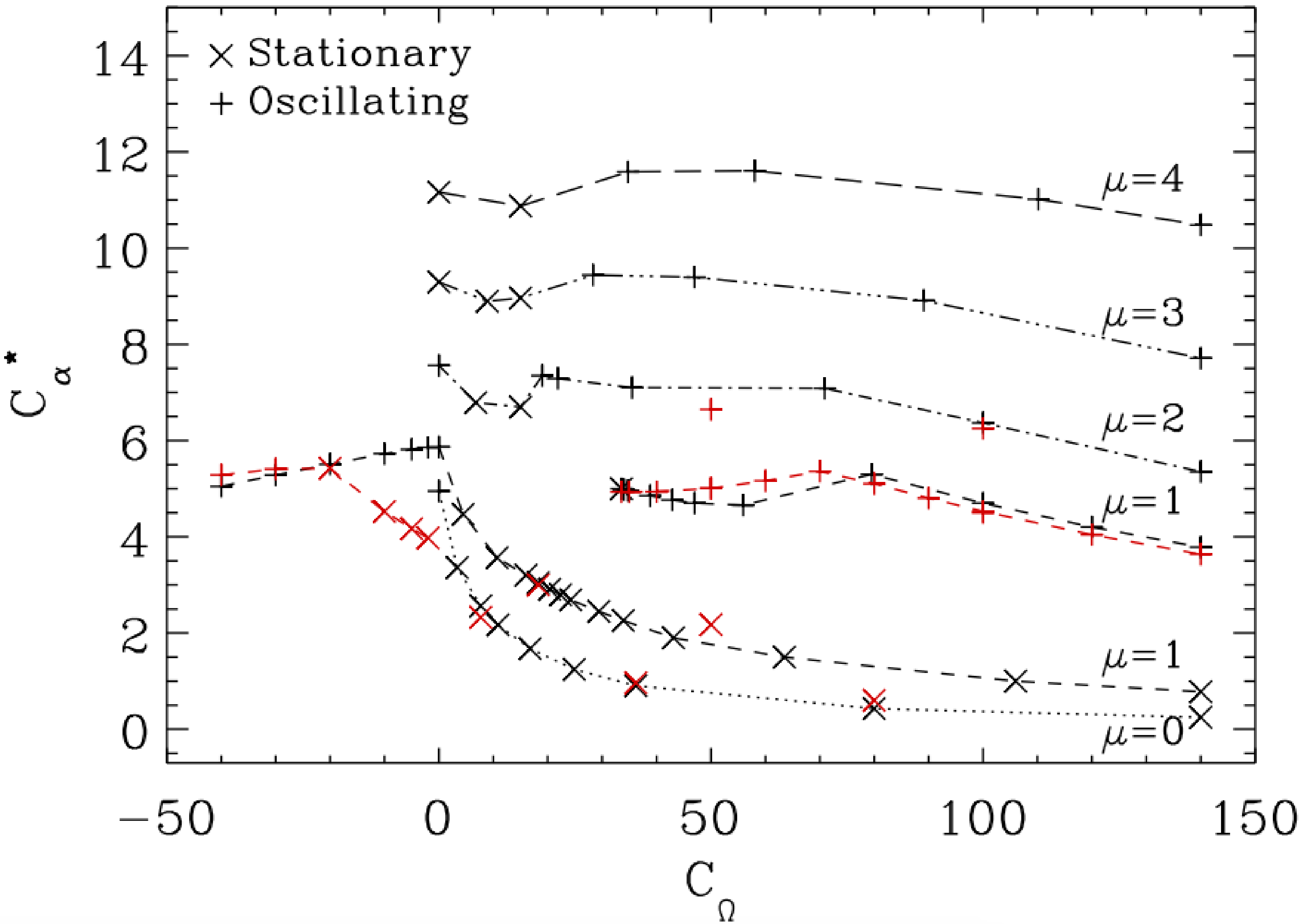
Addition of shear: $(S, \partial_\theta S, 0)$

Damping term

$$C_\alpha = \alpha_0 R / \eta_{\mathrm{t}0}$$

$$C_\Omega = S_0 R^2 / \eta_{\mathrm{t}0}$$

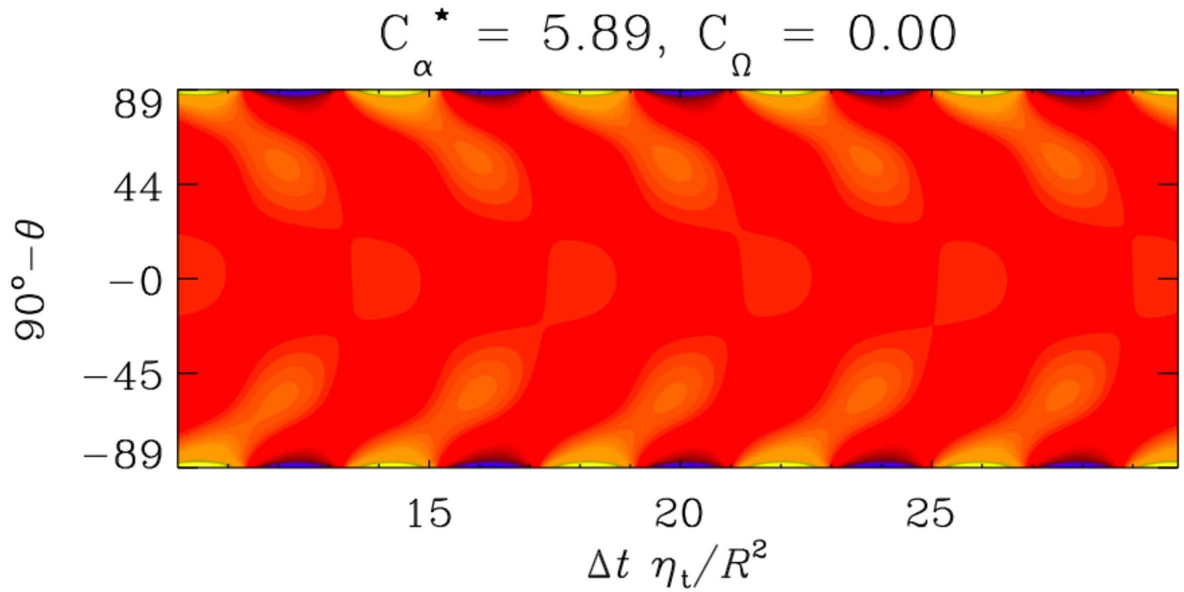
$\alpha^2\Omega$ Dynamos: adding shear



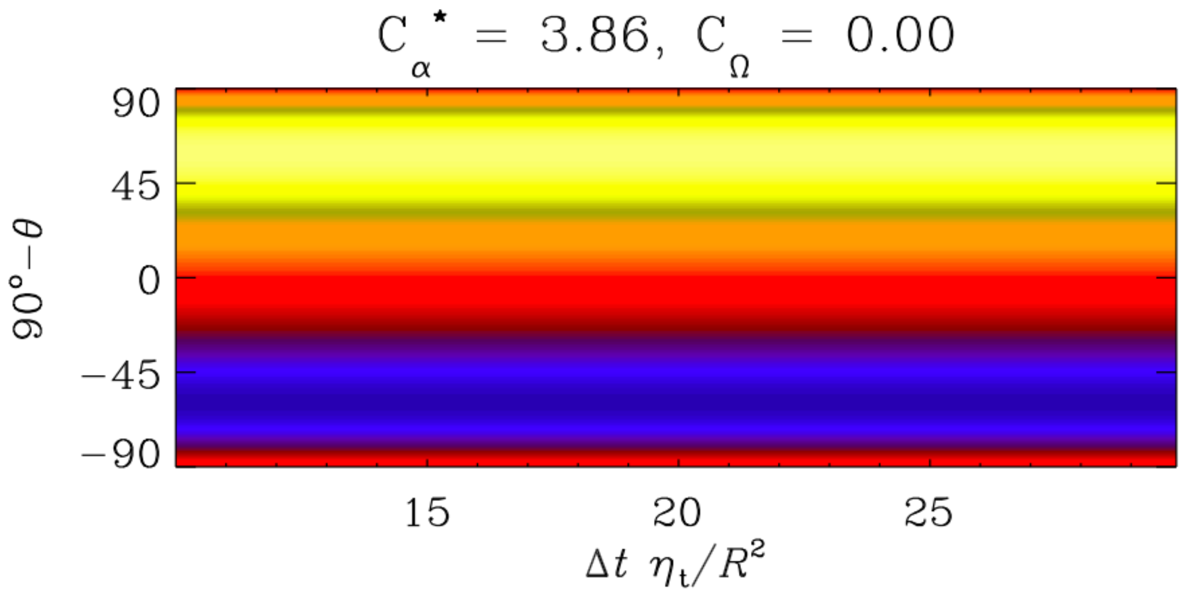
$\theta_0 = 1°$

$\theta_0 = 0°$

$\alpha^2\Omega$ Dynamos: adding shear

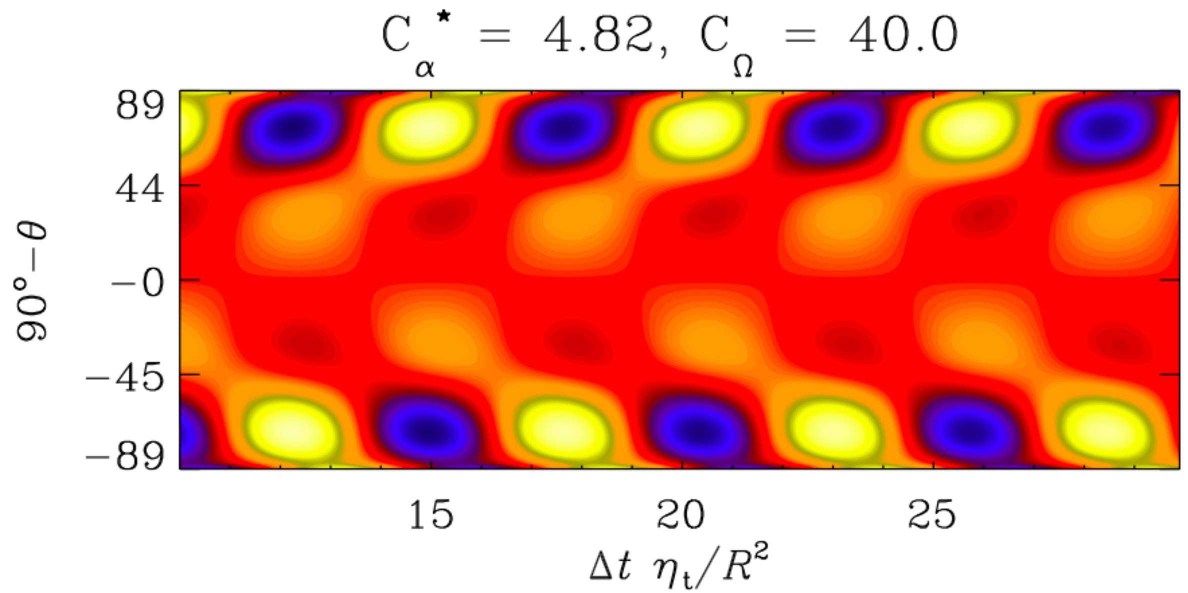


$\theta_0 = 1°$

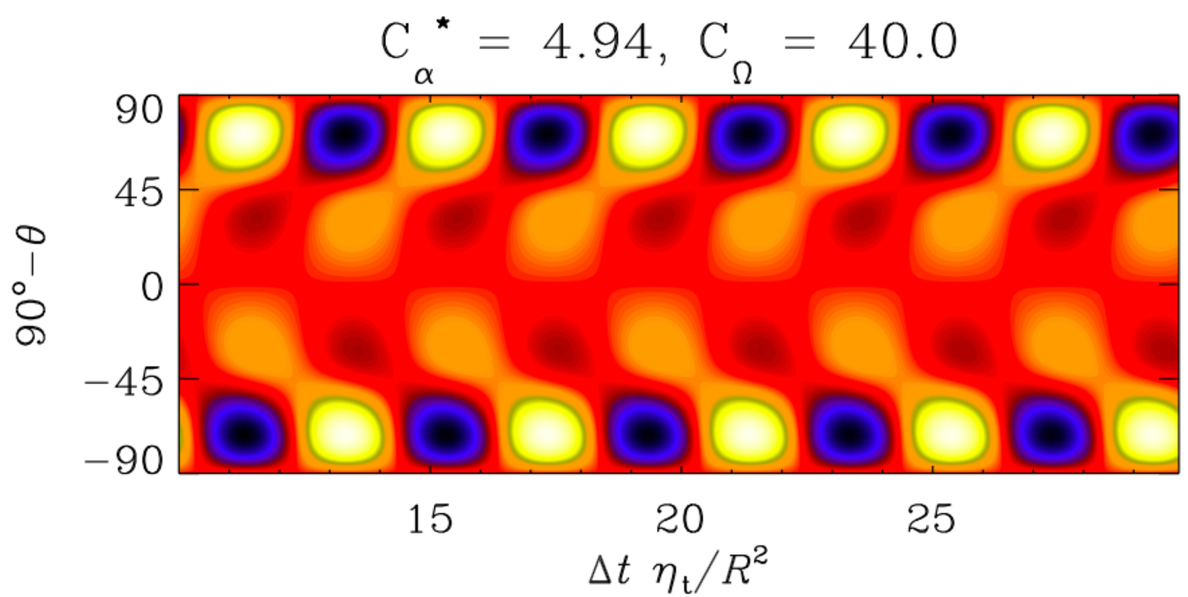


$\theta_0 = 0°$

$\alpha^2\Omega$ Dynamos: adding shear

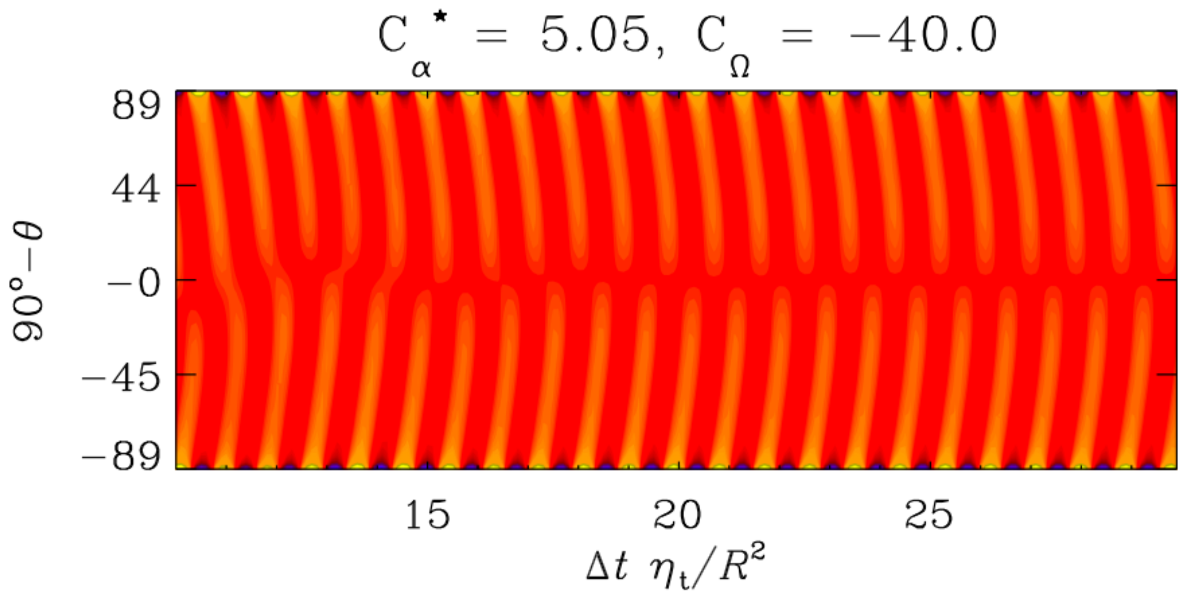


$\theta_0 = 1°$

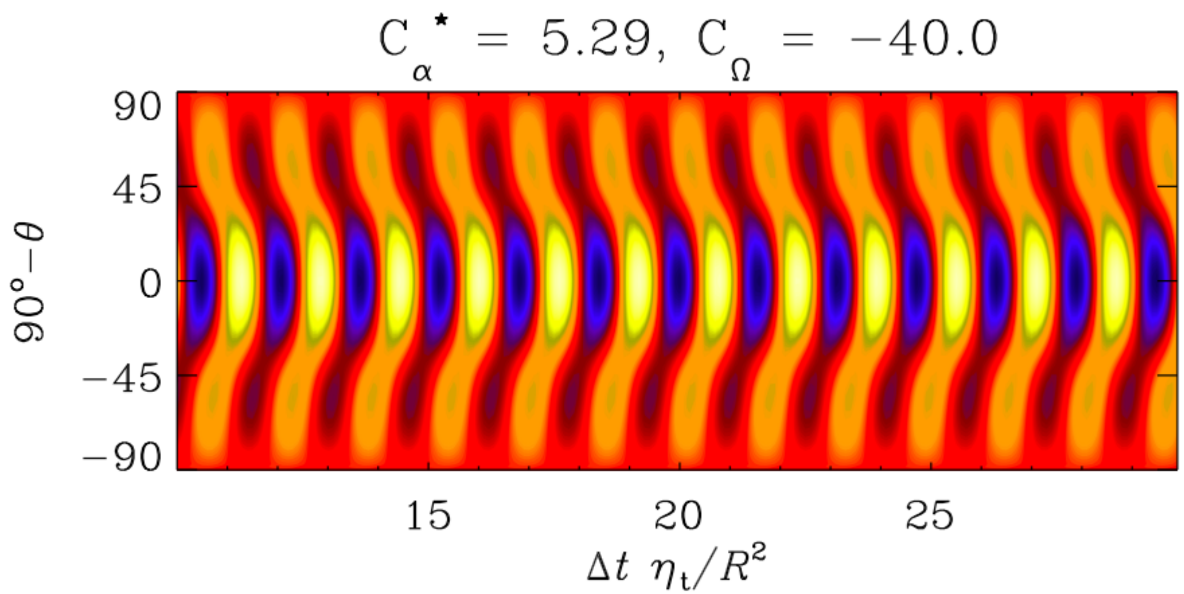


$\theta_0 = 0°$

$\alpha^2\Omega$ Dynamos: adding shear



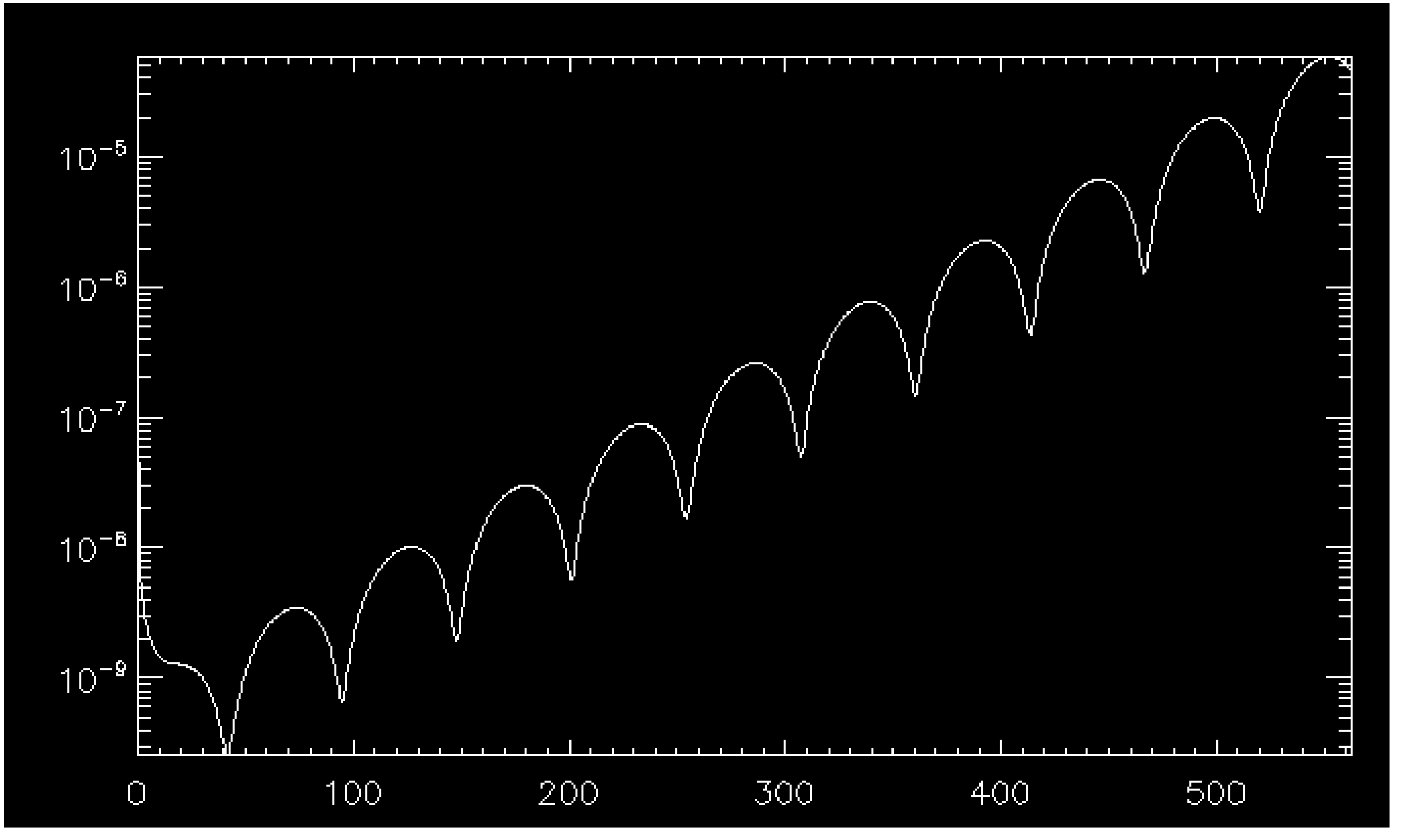
$\theta_0 = 1°$



$\theta_0 = 0°$

Problem:

When altering the conductivity profile, if η is too small, even in the absence of an α-effect, there is still growth



$\eta = 0.001$,
$\alpha = 0.$

α^2 Dynamos: memory effect?

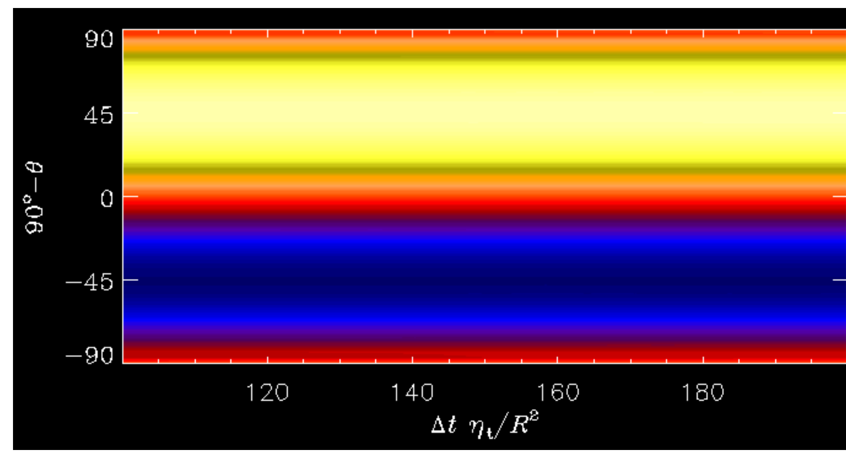
$$\overline{\boldsymbol{\mathcal{E}}} = \alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \mu_0 \overline{\boldsymbol{J}}$$

$$\downarrow$$

$$\frac{\partial \overline{\boldsymbol{\mathcal{E}}}}{\partial t} = \frac{1}{\tau} \left(\alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \mu_0 \overline{\boldsymbol{J}} - \overline{\boldsymbol{\mathcal{E}}} \right) + \eta_{\mathcal{E}} \nabla^2 \overline{\boldsymbol{\mathcal{E}}}$$

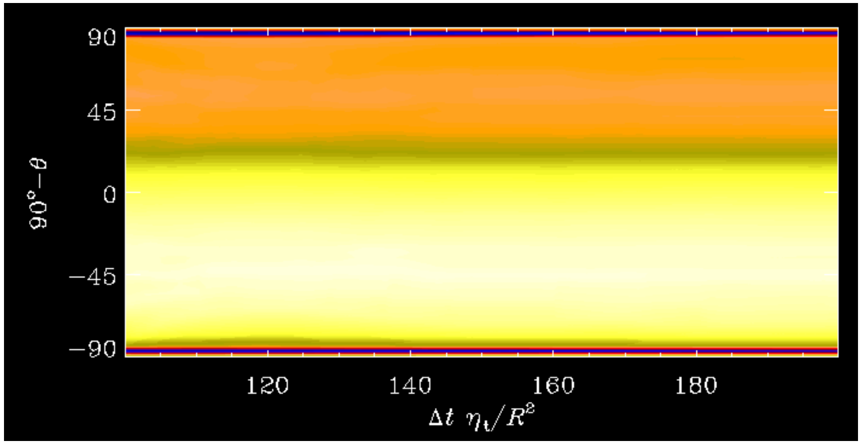
α^2 Dynamos: memory effect?

```
!  -*-f90-*-  (for Emacs)    vim:set filetype=fortran:  (for vim)
!
!
&run_pars
  nt=500000, it1=1000, isave=1000, itorder=3,ialive=1
  d2davg=0.1,dsnap=5e9, dvid=1e9,slice_position='w'
  lpencil_check_small=F
/
&magnetic_run_pars
  eta=.1, llorentzforce=F
/
&magn_mf_run_pars
  meanfield_etat=1., !meanfield_etat_profile='sin2y'
  alpha_effect=11.03, alpha_quenching=0., alpha_profile='cosy'
/
&magn_mf_demfdt_run_pars
  tau1_emf=.1, eta_emf_over_etat=1.
/
&special_run_pars
  !kf_alpm=60.    !(nonlinear saturation via helicity constraint)
  kf_alpm=0.      !(for linear theory put kf_alpm=0)
/
```



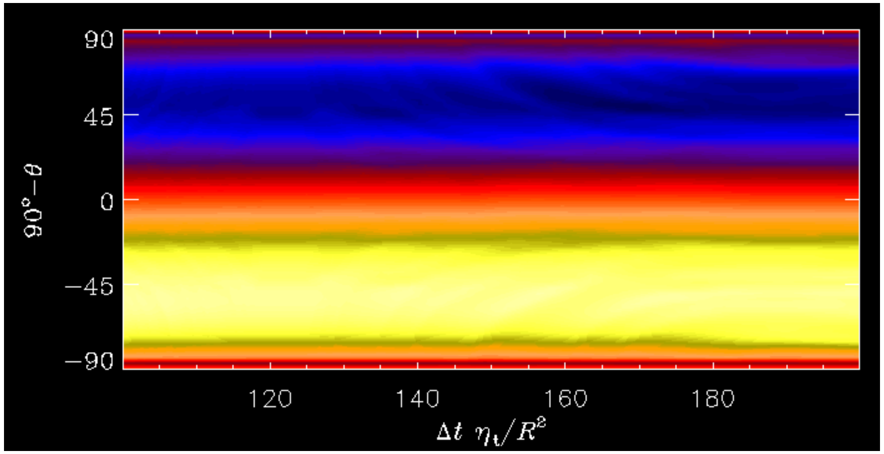
α^2 Dynamos: memory effect?

```
!  -*-f90-*-  (for Emacs)    vim:set filetype=fortran:  (for vim)
!
!
&run_pars
  nt=500000, it1=1000, isave=1000, itorder=3,ialive=1
  d2davg=0.1,dsnap=5e9, dvid=1e9,slice_position='w'
  lpencil_check_small=F
/
&magnetic_run_pars
  eta=.01, llorentzforce=F
/
&magn_mf_run_pars
  meanfield_etat=1., !meanfield_etat_profile='sin2y'
| alpha_effect=3.964, alpha_quenching=0., alpha_profile='cosy'
/
&magn_mf_demfdt_run_pars
  tau1_emf=.1, eta_emf_over_etat=1.
/
&special_run_pars
  !kf_alpm=60.   !(nonlinear saturation via helicity constraint)
  kf_alpm=0.     !(for linear theory put kf_alpm=0)
/
```



α^2 Dynamos: memory effect?

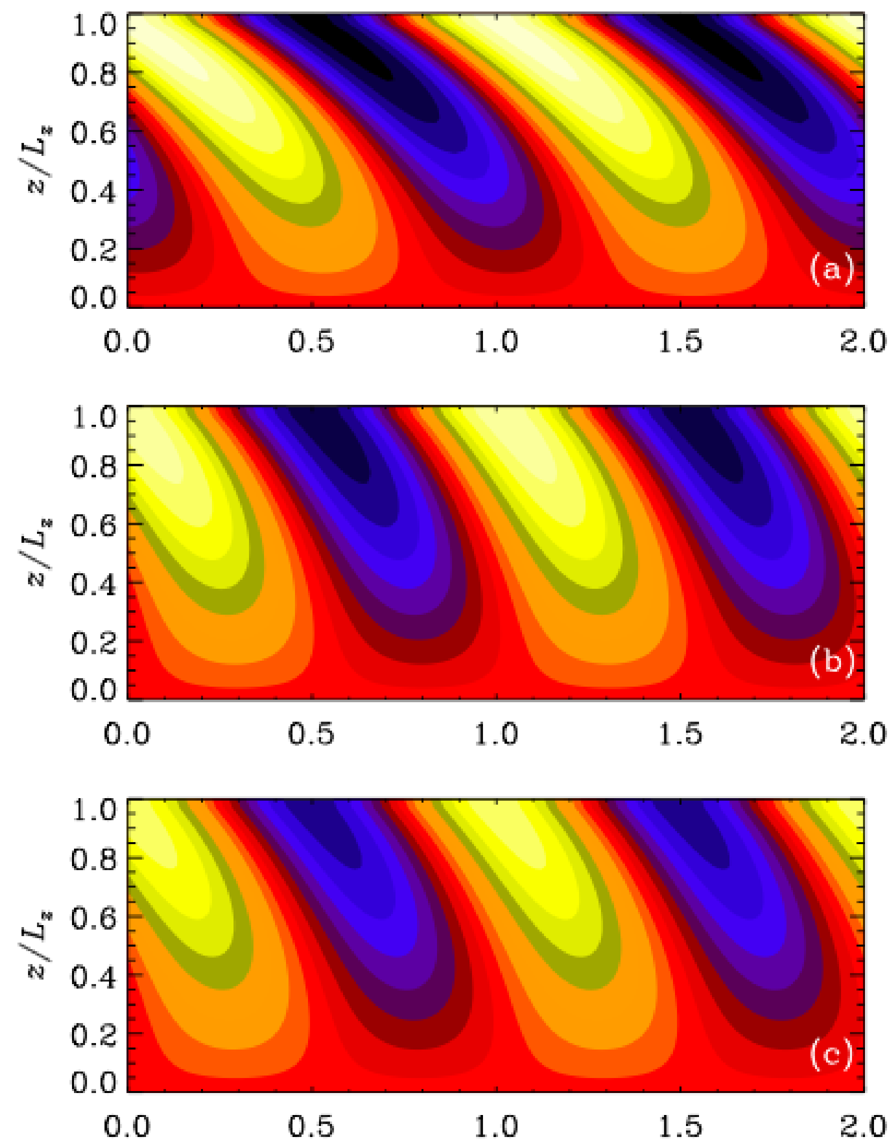
```
!  -*-f90-*-  (for Emacs)    vim:set filetype=fortran:  (for vim)
!
!
&run_pars
  nt=500000, it1=1000, isave=1000, itorder=3,ialive=1
  d2davg=0.1,dsnap=5e9, dvid=1e9,slice_position='w'
  lpencil_check_small=F
/
&magnetic_run_pars
  eta=.001, llorentzforce=F
/
&magn_mf_run_pars
  meanfield_etat=1., !meanfield_etat_profile='sin2y'
  alpha_effect=3.025, alpha_quenching=0., alpha_profile='cosy'
/
&magn_mf_demfdt_run_pars
  tau1_emf=.1, eta_emf_over_etat=1.
/
&special_run_pars
  !kf_alpm=60.   !(nonlinear saturation via helicity constraint)
  kf_alpm=0.     !(for linear theory put kf_alpm=0)
/
```



α^2 Dynamos: memory effect?

Rheinhardt & Brandenburg, 2011

Run	$\tau\eta_t k_1^2$	η_ε/η_t	C_α^{crit}	$\omega/\eta_t k_1^2$
(a)	0.001	0.001	5.16	1.64
	0.1	0.001	4.65	0.74
(b)	1	0.001	2.76	0.88
	1	0.1	2.77	0.87
(c)	1	0.3	2.84	0.86
	1	0.7	3.68	0.78
(d)	1	1	5.30	0.64
(e)	0.06	3	8.12	0.58



α^2 Dynamos: memory effect?

Run	$\tau\eta_t k_1^2$	η_ε/η_t	C_α^{crit}	$\omega/\eta_t k_1^2$
(a)	0.001	0.001	5.16	1.64
	0.1	0.001	4.65	0.74
(b)	1	0.001	2.76	0.88
	1	0.1	2.77	0.87
(c)	1	0.3	2.84	0.86
	1	0.7	3.68	0.78
(d)	1	1	5.30	0.64
(e)	0.06	3	8.12	0.58