# Declarative Extension of SAT Solvers with New Propagators

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# Introduction

Several lines of work involve:

High-level declarative languages

for specifying search problems

(e.g. find a Hamiltonian Path in a graph)

plus

Solvers for use in practice

E.g.: Constraint Modelling Languages (Essence, Zinc, . . . )
ASP, IDP system, Enfragmo, NP-Spec, etc.

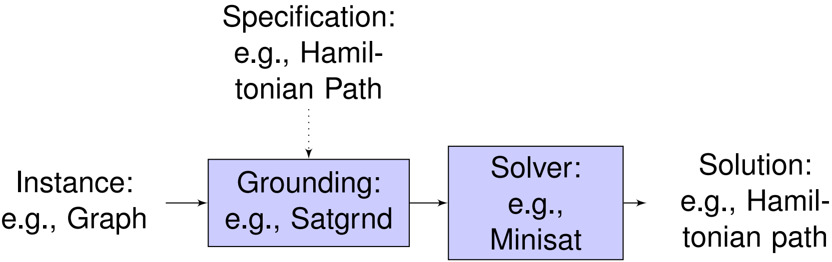
# Example: Hamiltonian Path Specification

The languages possess nice features: arithmetic, aggregates, induction(sometimes)
that allow one to write natural specifications of many problems

**Example:** FO+Arithmetic specification of Hamiltonian Path:

$$\forall u \forall v\ (next(u,v) \Rightarrow arc(u,v)).$$
$$\forall u \forall v \forall v'\ (next(u,v) \wedge next(u,v') \Rightarrow v = v').$$
$$\forall u \forall u' \forall v\ (next(u,v) \wedge next(u',v) \Rightarrow u = u').$$
$$\forall u\ (reach(0,u) \Leftrightarrow u = s).$$
$$\forall n\, \forall u\ (reach(n+1,u) \Leftrightarrow reach(n,u) \vee \exists v\ (reach(n,v) \wedge next(v,u))$$
$$\forall u\ (reach(|nodes|,u)).$$

# Existing Practice




So, a grounder takes the specification and an instance of a problem to generate a ground instance that is solved by a black-box solver

BUT, what if:

- the black-box solver gets stuck?
- or, solver's inference methods are too slow for our problem?

# Our Goals

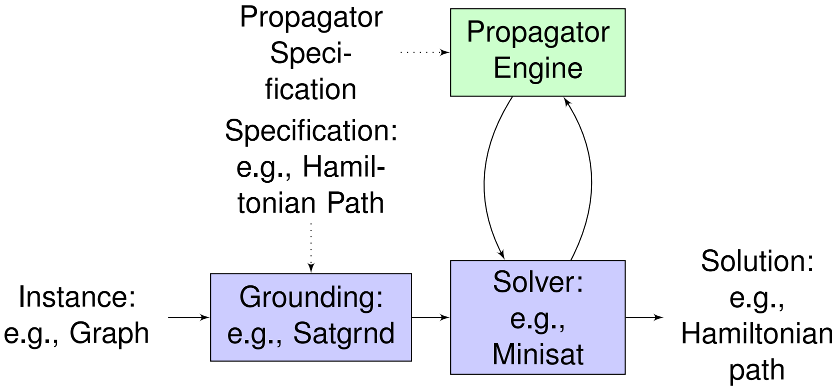
Our final long-term goal is to:

> Develop methods and foundations for declarative specification and automatic construction of optimized problem-specific solvers

In this paper, our goal is to:

> Develop methods to specify problem-specific propagators and to automatically modify a SAT solver to use such mechanisms

# Our Approach




So, we can:

- specify problem-specific propagators, and,
- incorporate those propagators into a solver

# Language $P[R]$ to Specify Problems Modulo Reasonings

Syntactically: $P[R]$ programs take the form of $(\phi, \{\psi_1, \ldots, \psi_n\})$ with $\phi$, $\psi_1$, …, and $\psi_n$ being first order programs
Here, $\phi$ is the main program and $\psi_1, \ldots, \psi_n$ are propagators

Propagator vocabulary of $\psi_i$ is $\tau_i = \mathit{vocab}(\psi_i) \setminus \mathit{vocab}(\phi)$

Semantically: $P[R]$ program $(\phi, \{\psi_1, \ldots, \psi_n\})$ is equivalent to following second-order formula:

$$\phi \wedge \bigwedge_{1 \leq i \leq n} \neg \exists \tau_i \, (\psi_i)$$

The role of propagators is to specify undesirable models.

# Example: Reachability Propagator for Hamiltonian Path

Main problem $\phi$ asserts that *next* is a subset of arcs *without any two arcs with the same source or the same destination*:

$$\phi := \left\{ \begin{array}{c} \forall x, y\ (next(x, y) \to arc(x, y)). \\ \forall x, y, y'\ (next(x, y) \wedge next(x, y') \to y = y'). \\ \forall x, x', y\ (next(x, y) \wedge next(x', y) \to x = x'). \\ \forall x\ (\neg next(x, s)). \end{array} \right\}$$

Propagator $\psi$ checks if a cut separates *s* from some other node

$$\psi := \left\{ \begin{array}{c} cut(s). \qquad \exists x\ \neg cut(x). \\ \forall x, y\ (next(x, y) \wedge cut(x) \to cut(y)). \end{array} \right\}$$

*P*[*R*] specification $(\phi, \{\psi\})$ finds Hamiltonian paths because propagator $\psi$ guarantees reachability of all nodes from *s*

# Grounding $P[R]$ Specifications

Underapproximate translation of propagator $\psi$, denoted as $[\![\psi]\!]^\varepsilon$, is formula $\psi'$ so that

- Positive occurrences of $R \in \varepsilon$ is replaced by $R_l$
- Negative occurrences of $R \in \varepsilon$ is replaced by $R_u$

Grounding of $P[R]$ specification $(\phi, \{\psi_1, \ldots, \psi_n\})$ w.r.t. $\mathcal{A}$, denoted by $Gnd((\phi, \{\psi_1, \ldots, \psi_n\}); \mathcal{A})$, is

$$(Gnd(\phi; \mathcal{A}), \{Gnd([\![\psi_1]\!]^\varepsilon; \mathcal{A}), \ldots, Gnd([\![\psi_n]\!]^\varepsilon; \mathcal{A})\})$$

where $\varepsilon = vocab(\phi) \setminus vocab(\mathcal{A})$

# Theoretical Foundations

If $\mathcal{B}$ is a $(\sigma \cup \varepsilon)$-structure that partially interprets $\varepsilon$, its 2-valued representation, denoted as $\mathcal{B}_{2v}$, is a $(\sigma \cup \varepsilon_l \cup \varepsilon_u)$-structure so that

- For $R \in \sigma$, $R^{\mathcal{B}_{2v}} = R^{\mathcal{B}}$,
- For $R \in \varepsilon$, $R_l^{\mathcal{B}_{2v}}$ is the lowerbound of $R^{\mathcal{B}}$, and $R_u^{\mathcal{B}_{2v}}$ is the upperbound of $R^{\mathcal{B}}$, i.e.,

$$\begin{aligned} R(\bar{x}) = \mathit{true} &\Rightarrow R_l(\bar{x}) = R_u(\bar{x}) = \mathit{true} \\ R(\bar{x}) = \mathit{unknown} &\Rightarrow R_l(\bar{x}) = \mathit{false}, R_u(\bar{x}) = \mathit{true} \\ R(\bar{x}) = \mathit{false} &\Rightarrow R_l(\bar{x}) = R_u(\bar{x}) = \mathit{false} \end{aligned}$$

Theorem: For partial str. $\mathcal{B}$:
$\mathcal{B}_{2v} \models \exists \tau \, [\![\psi_i]\!]^{\varepsilon} \Rightarrow$ all extensions of $\mathcal{B}$ falsify $(\phi, \{\psi_1, \ldots, \psi_n\})$

# Example: Hamiltonian Path

Underapproximate translation of reachability propagator $\psi$ is

$$[\![\psi]\!]^{\{next\}} := \left\{ \begin{array}{c} cut(s). \qquad \exists x \neg cut(x). \\ \forall x, y\ (next_u(x, y) \wedge cut(x) \rightarrow cut(y)). \end{array} \right\}$$

Here, if $\mathcal{B}_{2v} \models \exists cut\ ([\![\psi]\!]^{\{next\}}) \Rightarrow$ (no matter how many unknown arcs in $next^{\mathcal{B}}$ are made true) $s$ cannot reach all nodes using $next^{\mathcal{B}}$

# Solving $P[R]$ Programs: SAT-to-SAT

For a given propagator $\psi$, we want to incorporate $\psi$ into a SAT solver for the main problem $\phi$

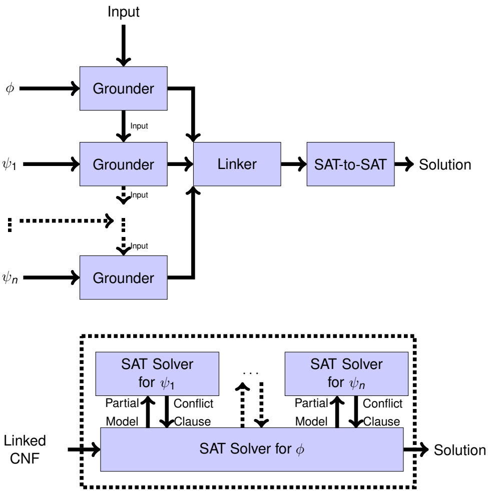
Note that a SAT solver state can be viewed as a partial structure

$\Rightarrow$ (by previous theorem) In state $\mathcal{B}$ of SAT solver for $\phi$, if $\mathcal{B}_{2v}$ satisfies $\exists\tau\ [\![\psi]\!]^{\varepsilon}$ then SAT solver for $\phi$ should backtrack (because there cannot be a model in this branch of search)

Also, note that, using a (new) SAT solver, we can check if a state $\mathcal{B}$ satisfies some propagator specification

$\Rightarrow$ The idea is to have a SAT solver for $\phi$ that communicates with other SAT solvers for $\psi_i$'s (hence the name SAT-to-SAT)

# $P[R]$ + SAT-to-SAT: In a Glance



# SAT-to-SAT: Conflict Clauses and Triggers

Implemented a propagator that, given state $\mathcal{B}$ of external solver:

- Computes 2-valued representation $\mathcal{B}_{2v}$ of $\mathcal{B}$,
- Runs an internal SAT solver to check if $\mathcal{B}_{2v} \models \exists\tau \; [\![\psi_i]\!]^{\varepsilon}$,
- If so, it returns $\langle SAT, J\rangle$ with $J$ a set of literals such that $\mathcal{B}' \models_3 \bigwedge_{x\in J} x \Rightarrow \mathcal{B}'_{2v} \models \exists\tau \; [\![\psi_i]\!]^{\varepsilon}$, and
- If not, it returns $\langle UNSAT, J\rangle$ with $J$ a set of literals such that $\mathcal{B}' \not\models_3 \bigvee_{x\in J} x \Rightarrow \mathcal{B}'_{2v} \not\models \exists\tau \; [\![\psi_i]\!]^{\varepsilon}$

Lazy conflict clause generation: If internal solver returned $\langle SAT, J\rangle$, add clause $\bigvee_{x\in J} \neg x$ to external solver
Triggers: If internal solver returned $\langle UNSAT, J\rangle$, do not run it again until external solver has assigned some $x \in J$ to true

Theorem: SAT-to-SAT algorithm with clause generation and triggers as above is sound and complete for $P[R]$ specifications

# Example: Hamiltonian Path with Reachability

For propagator $\psi$ in Hamiltonian path:

Conflict clauses always take the following form:

$$\bigvee_{u \in cut, v \notin cut, arc(u,v)} next(u, v)$$

where

- *cut* is a proper subset of nodes including starting node *s*, and
- $next(u, v)$ is assigned to false whenever $u \in cut$ and $v \notin cut$

Triggers will always be of the form
$T = \{\neg next(u_1, v_1), \ldots, \neg next(u_k, v_k)\}$
where $T$ contains a rooted spanning tree of $\mathcal{G}$ with root $s$

# Experiments: Hamiltonian Path

| Hamiltonian Path Instances (15m time limit) | | | | | | | |
|---|---|---|---|---|---|---|---|
| Size | Total Inst. | Glucose Direct Enc. | | SAT-to-SAT | | | |
| | | | | Reachability Enc. | | Acyclicity Enc. | |
| | | # | Time | # | Time | # | Time |
| 50 | 20 | 20 | 4.85s | **20** | **0.02s** | **20** | **0.02s** |
| 100 | 20 | 4 | 390s | **20** | **0.13s** | 20 | 0.63s |
| 150 | 20 | 0 | — | **20** | **1.14s** | 20 | 7.52s |
| 200 | 20 | 0 | — | **20** | **9.00s** | 20 | 74.0s |
| 250 | 20 | 0 | — | **20** | **82.3s** | 18 | 283s |
| 300 | 20 | 0 | — | **9** | **288s** | 5 | 639s |

Table: Solving Hamiltonian path using SAT-to-SAT on two different encodings plus using Glucose on a direct encoding.

# Future Directions

Upcoming soon (with Bart and Tomi):

- Solving QBF: i.e., arbitrary levels of nesting in SAT-to-SAT
- Generating KR solvers from their semantics: i.e., a second-order front-end for SAT-to-SAT

And, later:

- Developing methods to reason about a solver's state and history, e.g., decision variables and their levels or propagated variables and their reasons
- Developing foundations of temporal reasoning about solving process

# Thank You

Questions?

# Example: Acyclicity Propagator for Hamiltonian Path

*P*[*R*] Specification $(\phi', \{\psi'\})$ also specifies Hamiltonian paths:

$$\phi' := \left\{ \begin{array}{l} \forall x, y\ (next(x, y) \rightarrow arc(x, y)). \\ \forall x, y, y'\ (next(x, y) \wedge next(x, y') \rightarrow y = y'). \\ \forall y\ (y \neq s \rightarrow \exists x\ (next(x, y))). \end{array} \right\}$$

$$\psi' := \left\{ \begin{array}{l} \forall y\ (cycle(y) \rightarrow \exists x\ (cycle(x) \wedge next(x, y))). \\ \exists x\ cycle(x). \end{array} \right\}$$

Here, propagator $\psi'$ checks if *next* has become cyclic
$(\phi', \{\psi'\})$ specifies Hamiltonian Paths because *next* has to be an acyclic path of size $n - 1$