Distributed Quantum Advantage for Local Problems

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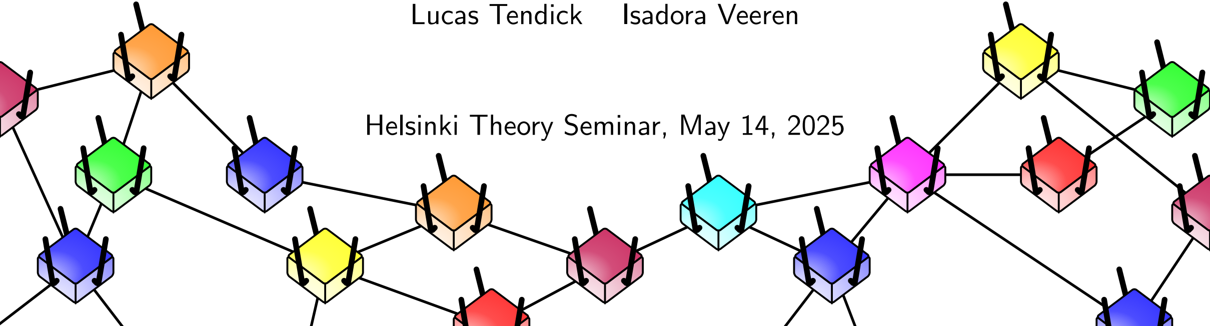
Massimo Equi   François Le Gall   **Henrik Lievonen**   Augusto Modanese
Dennis Olivetti   Marc-Olivier Renou   Jukka Suomela
Lucas Tendick   Isadora Veeren

Helsinki Theory Seminar, May 14, 2025



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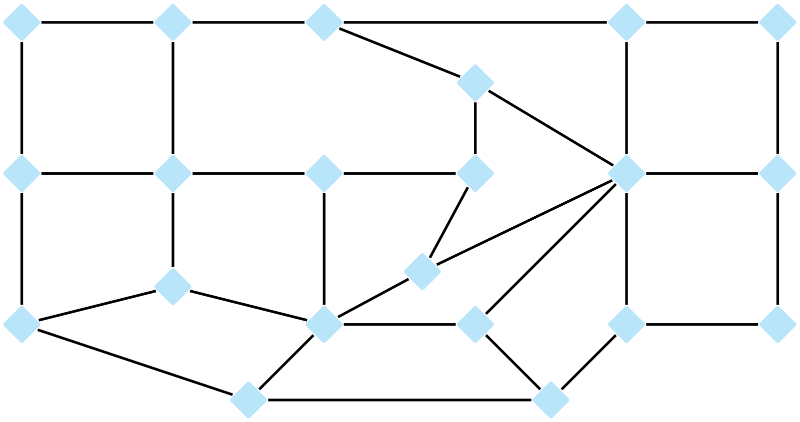
LOCAL model



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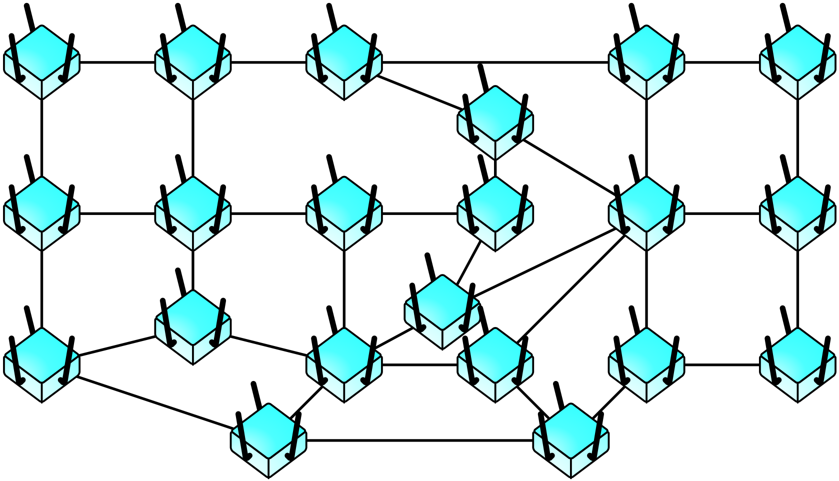

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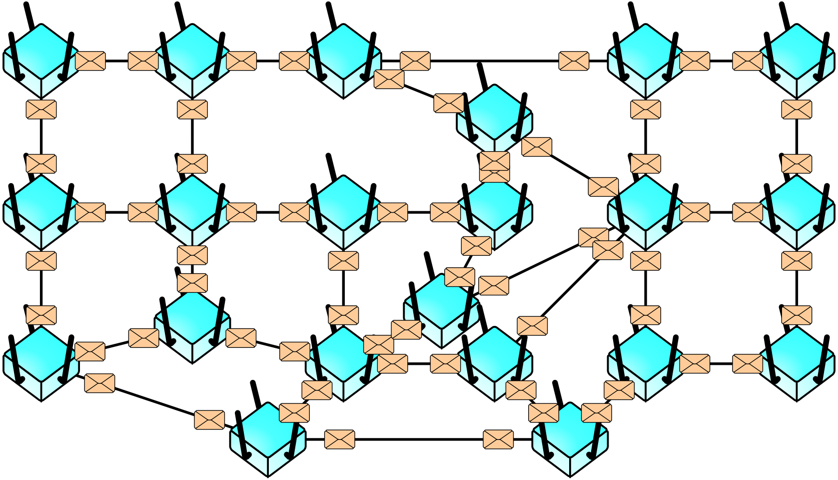

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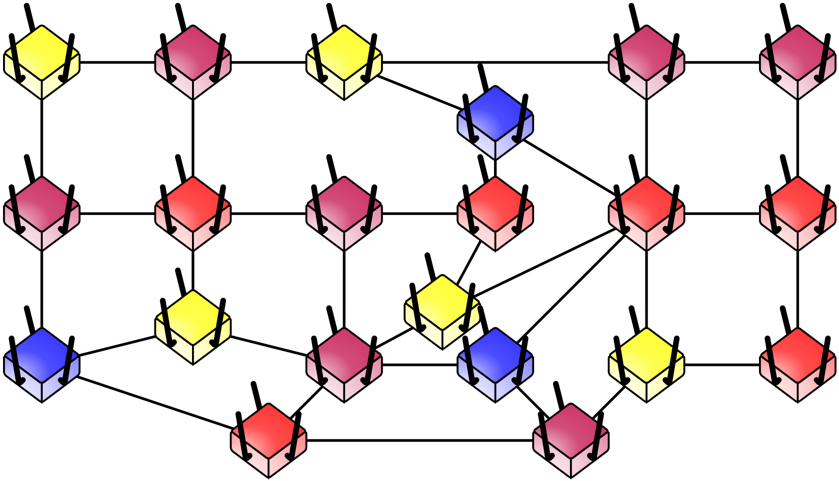

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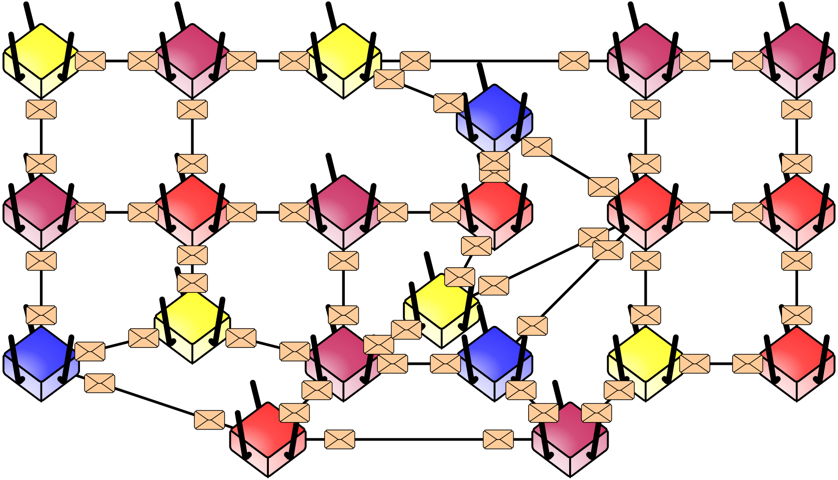

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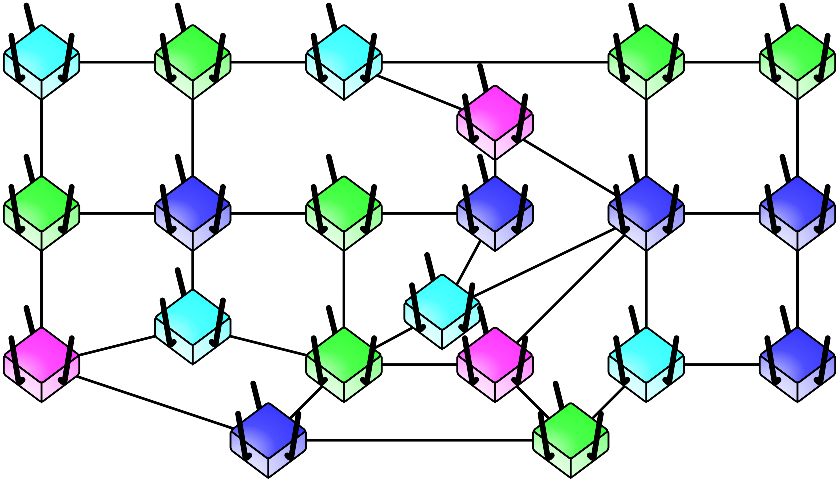

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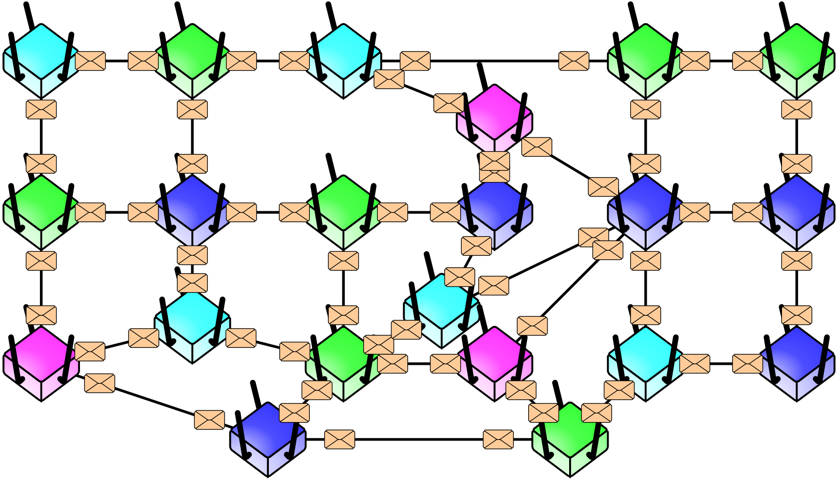

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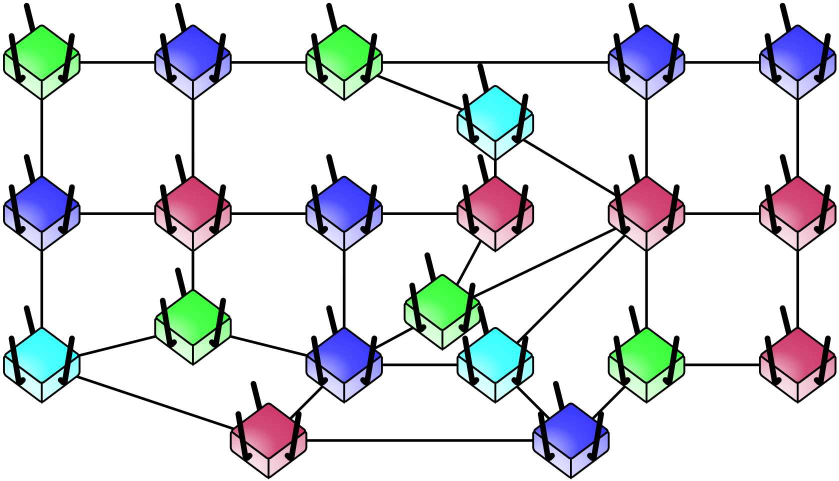

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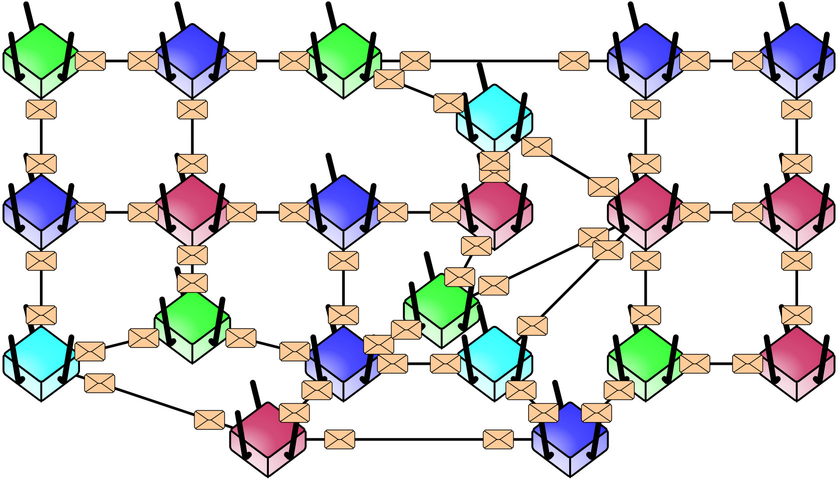

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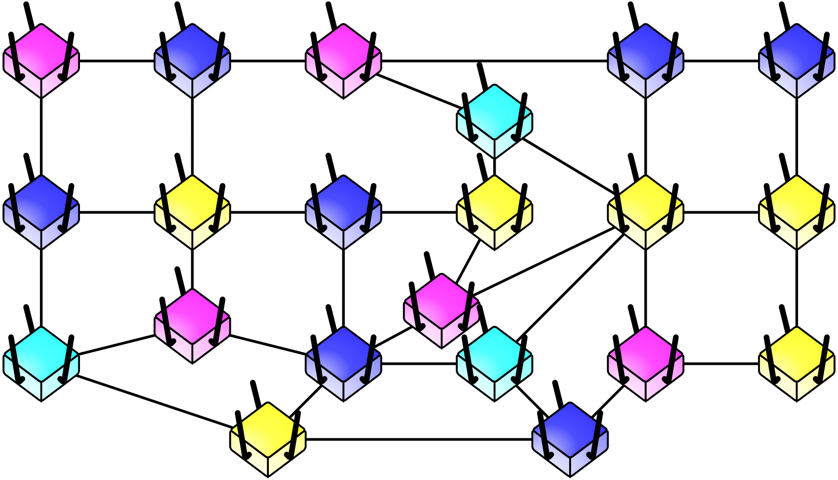
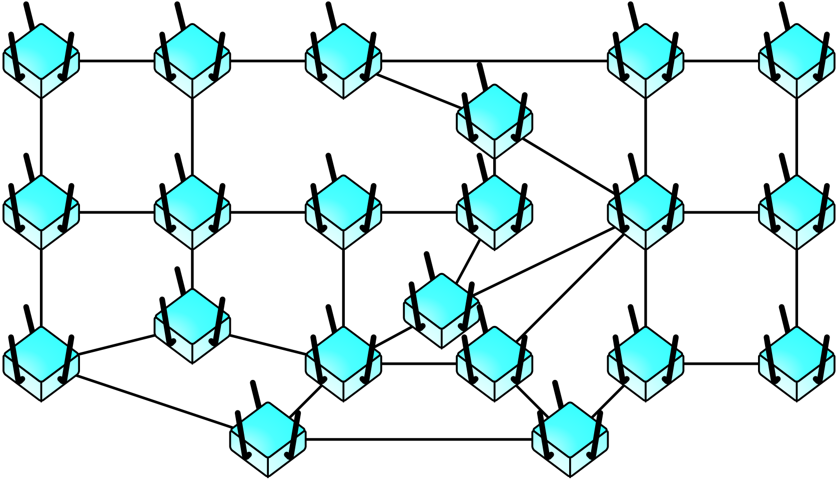


Quantum-LOCAL model



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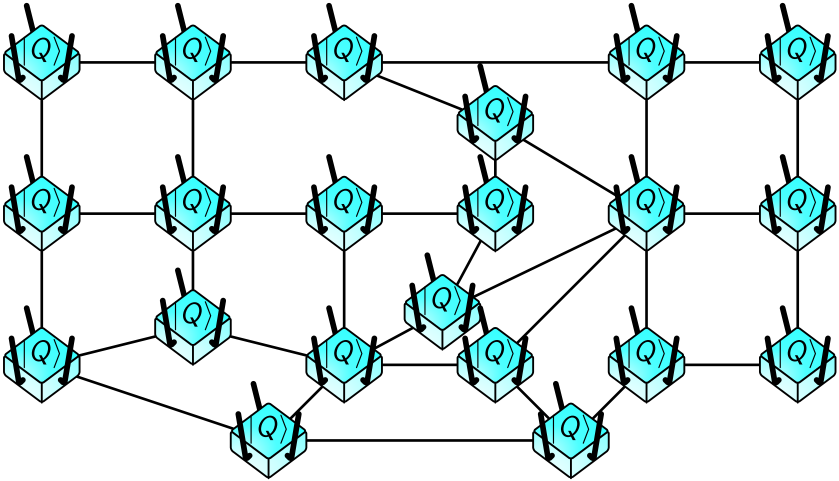

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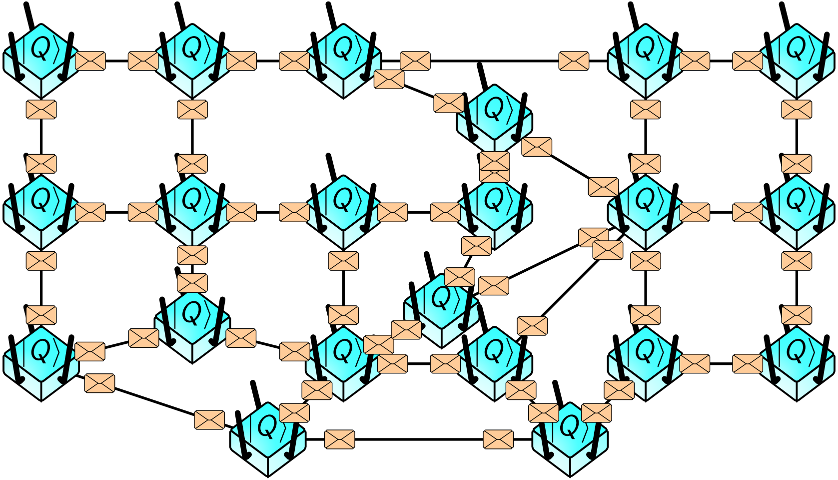

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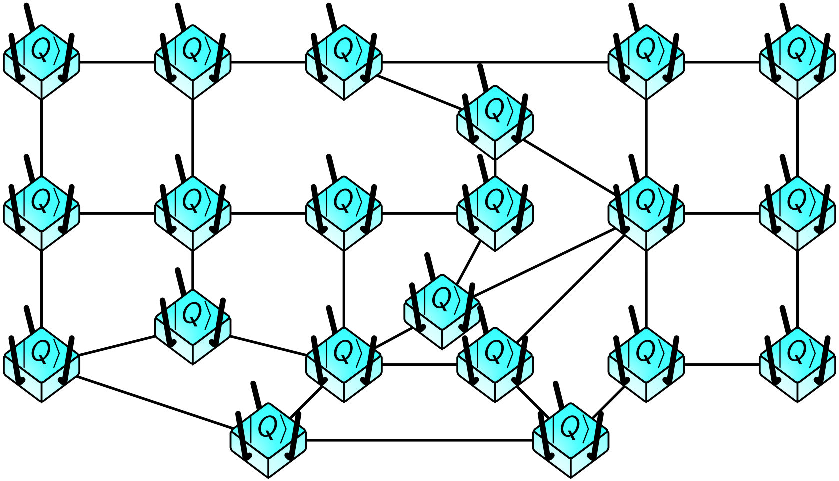

Previous work

Theorem (Le Gall, Nishimura, and Rosmanis 2019)

There exists a global computational problem that can be solved with 2 round in the quantum-LOCAL model, but requires $\Omega(n)$ *rounds in the classical LOCAL model.*

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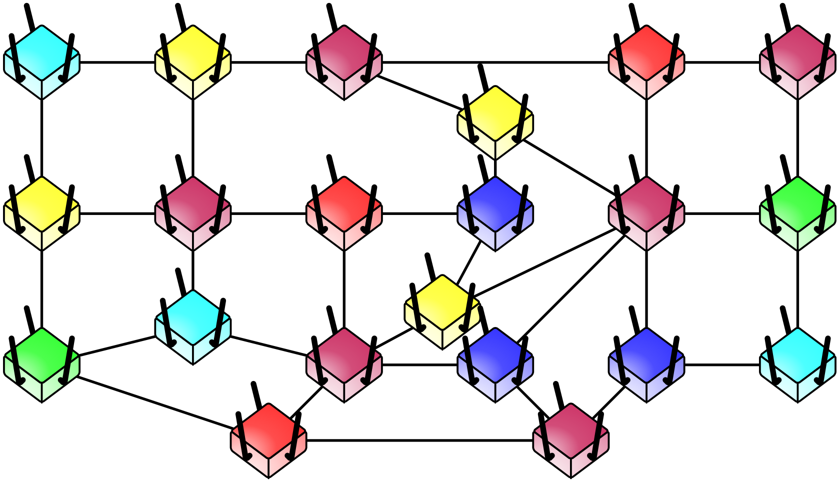
Distributed Quantum Advantage for Local Problems

Local problems



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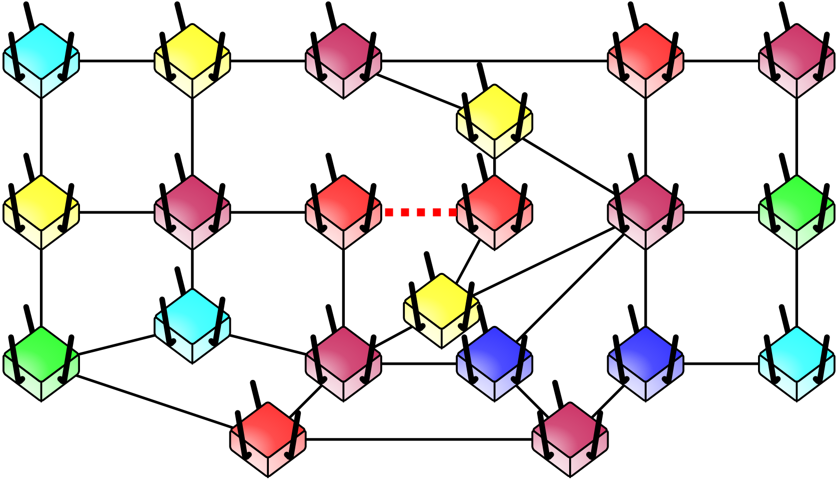

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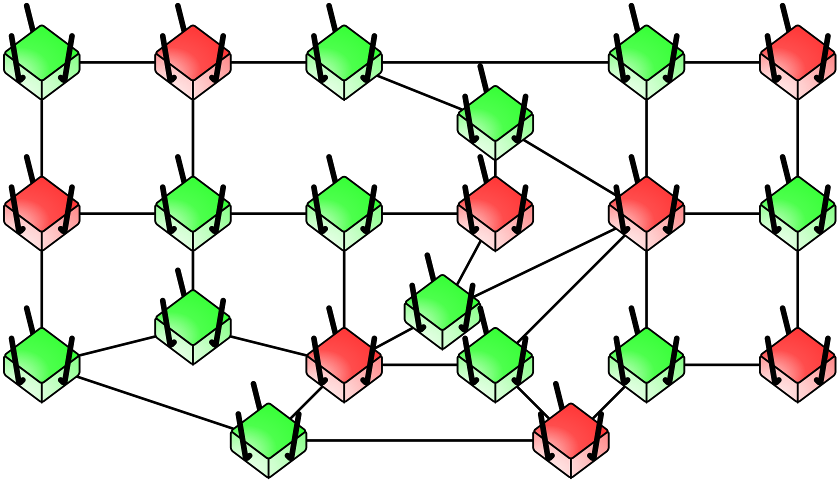

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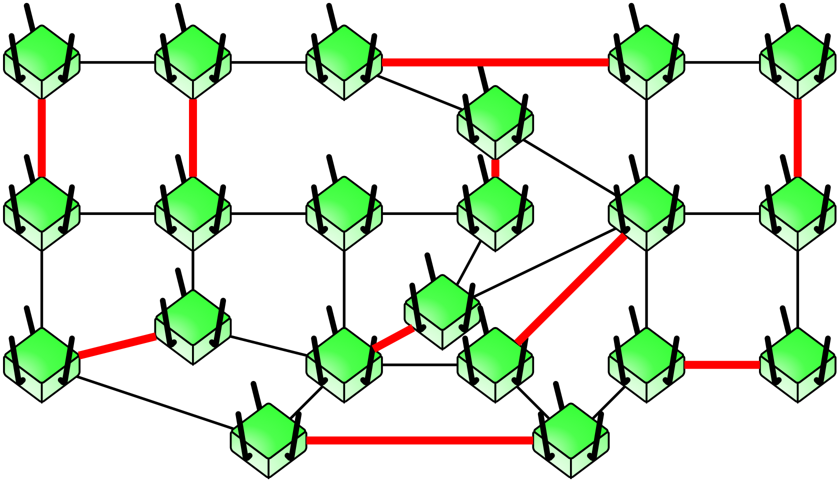

Distributed Quantum Advantage for Local Problems

Games

Relation R

x	y	$a \oplus b$
0	0	$0 \mod 2$
0	1	$0 \mod 2$
1	0	$0 \mod 2$
1	1	$1 \mod 2$

Games

Alice

Bob

Relation R

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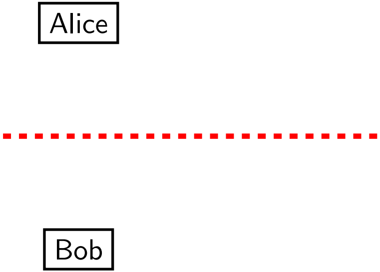

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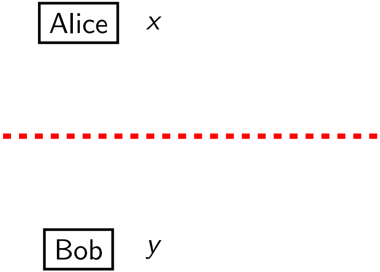

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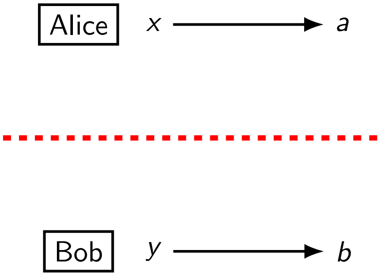
Games



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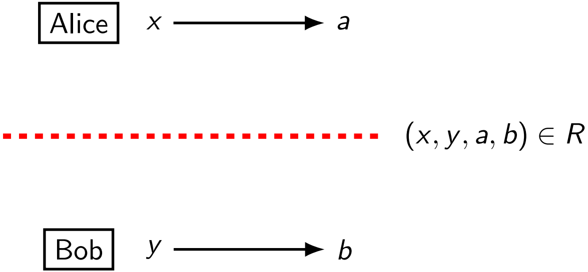

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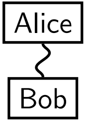

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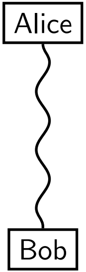

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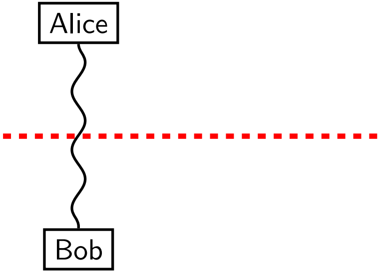

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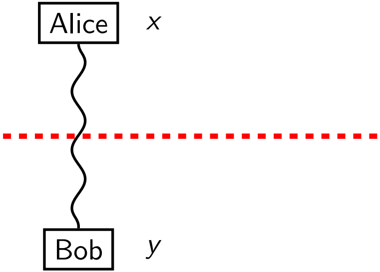
Quantum games



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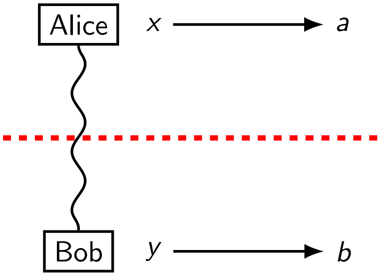
Quantum games



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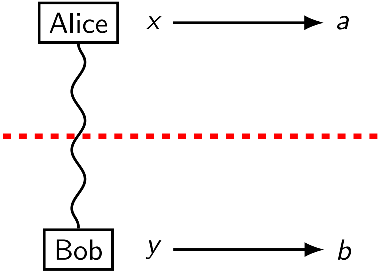

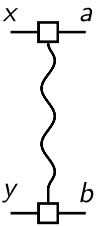

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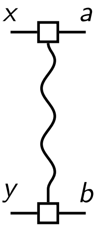

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CHSH[1] game

x	y	$a \oplus b$
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[1] Clauser, Horne, Shimony, and Holt 1969

CHSH[1] game

x	y	$a \oplus b$
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Winning probability:

Classic
75%

[1] Clauser, Horne, Shimony, and Holt 1969

CHSH[1] game

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Winning probability:

Classic	Quantum
75%	$\cos^2\left(\frac{\pi}{8}\right) \approx 85\%$

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CHSH[1] game

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Winning probability:

Classic	Quantum	Non-Signaling
75%	$\cos^2\left(\frac{\pi}{8}\right) \approx 85\%$	100%

[1] Clauser, Horne, Shimony, and Holt 1969

CHSH[1] game

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Winning probability:

Classic	Quantum	~~Non-Signaling~~ Super-Quantum
75%	$\cos^2\left(\frac{\pi}{8}\right) \approx 85\%$	100%

[1] Clauser, Horne, Shimony, and Holt 1969

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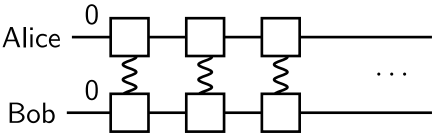
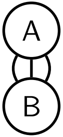
Networks of games





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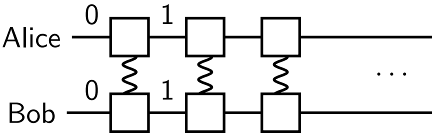

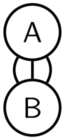

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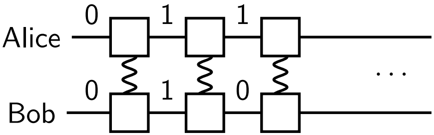

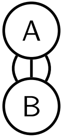

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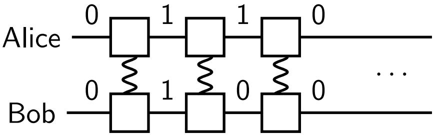

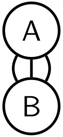

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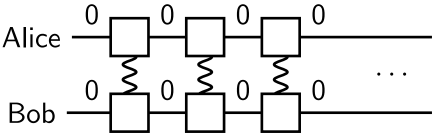

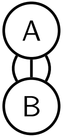

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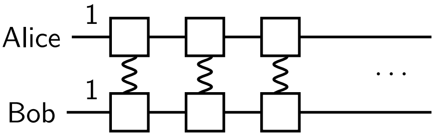

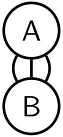

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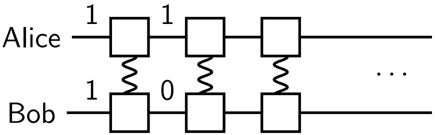

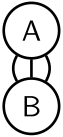

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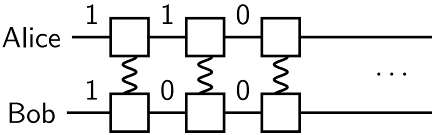

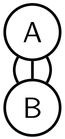

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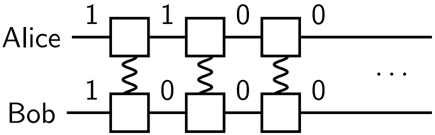

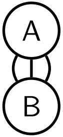

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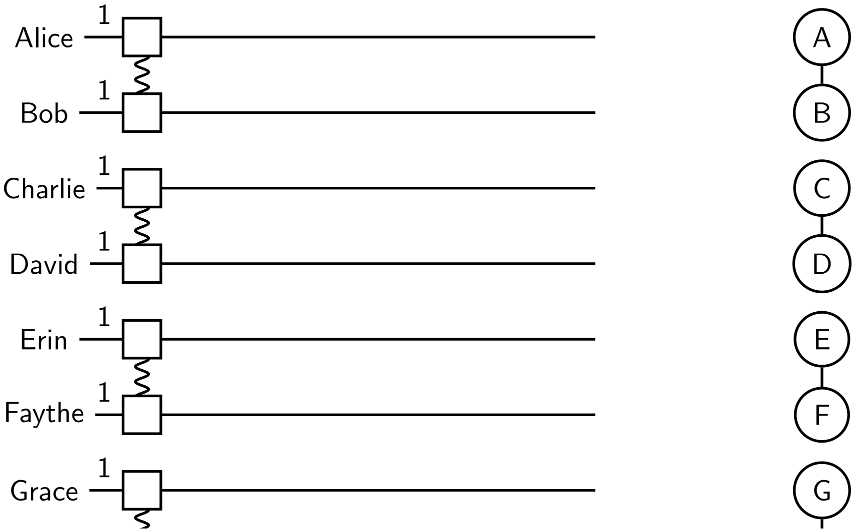

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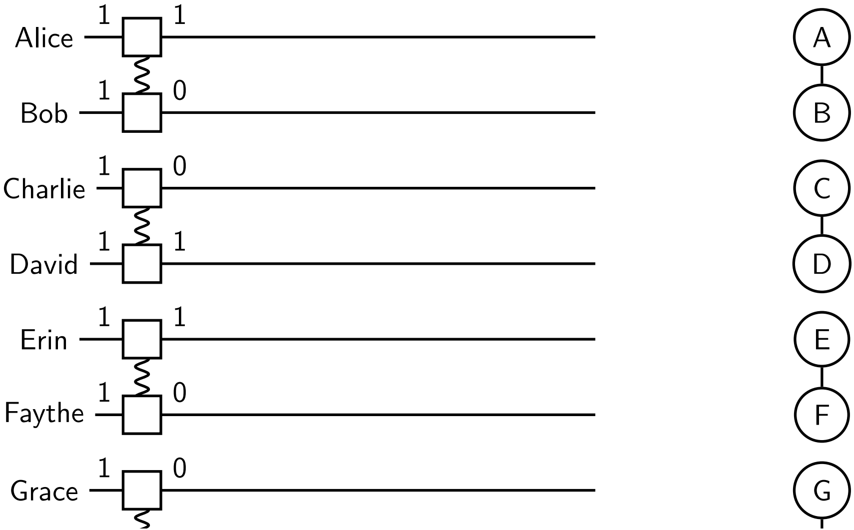

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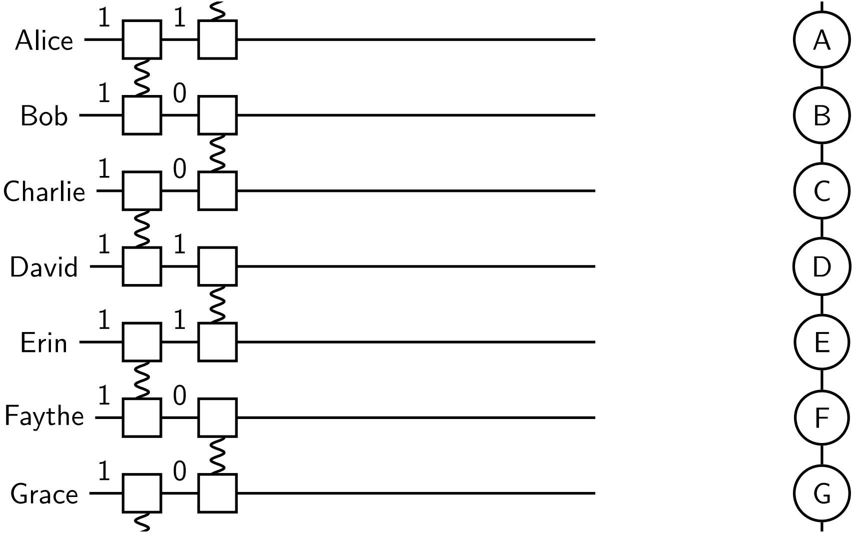

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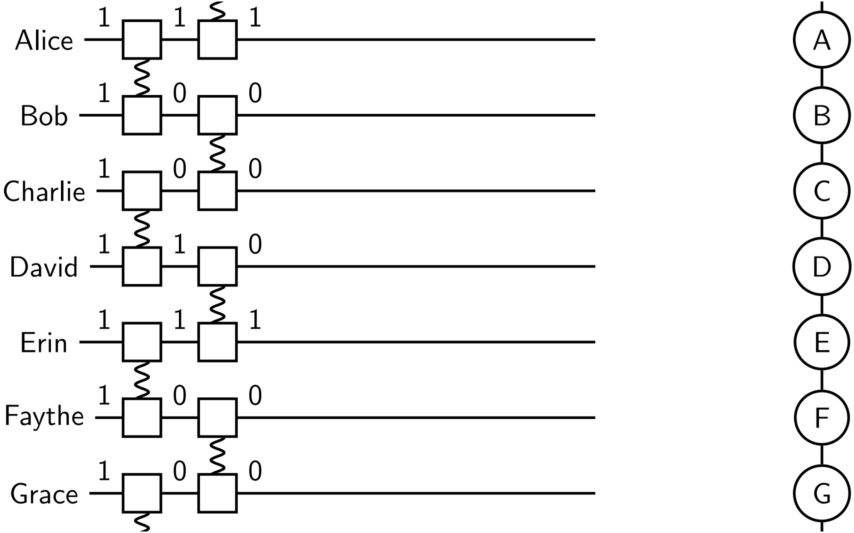

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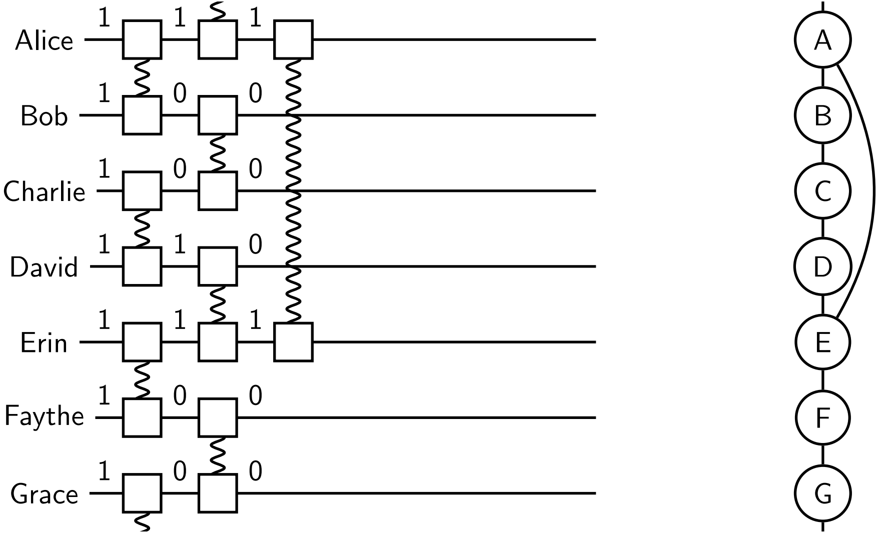

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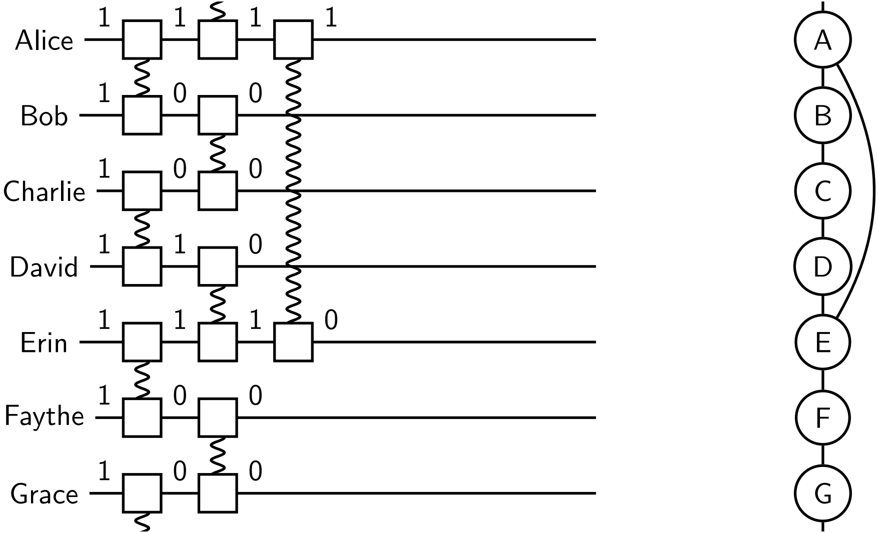

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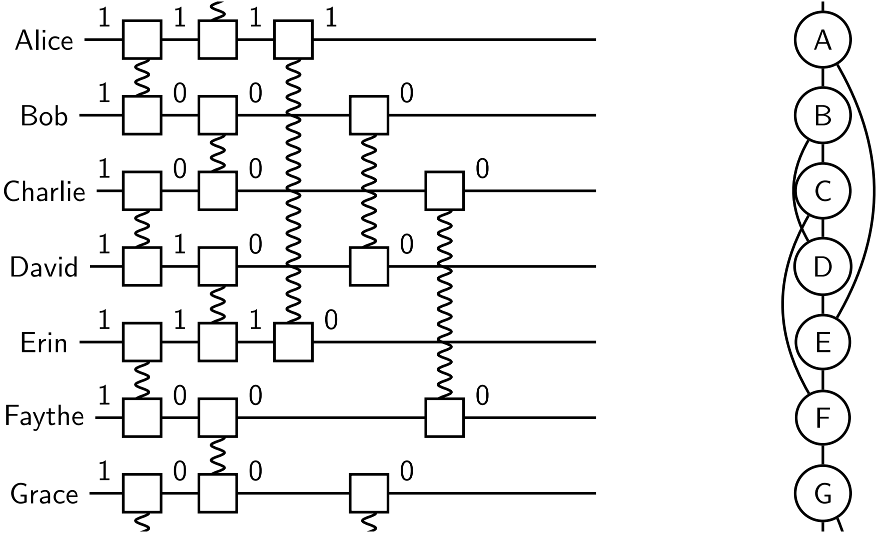

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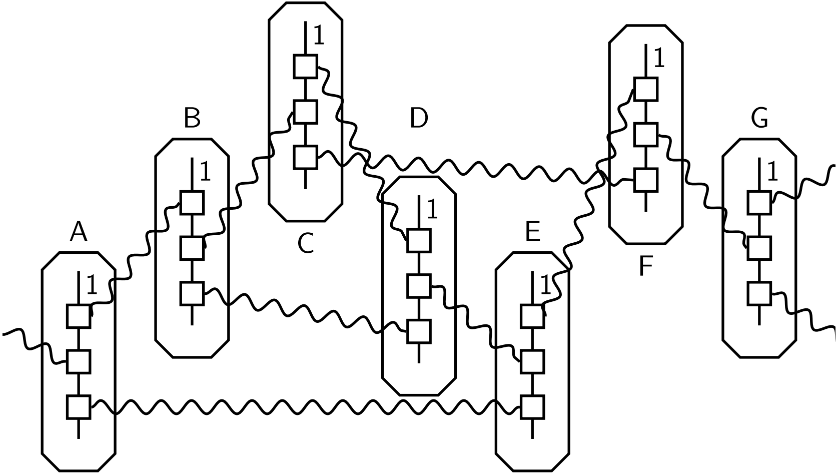

Summary

	Classic	Super-Quantum	Local
Previous work[2]	$\Omega(n)$	$O(1)$	No
Our work	$\Theta(\Delta)$	$O(1)$	Yes

[2] Le Gall, Nishimura, and Rosmanis 2019

GHZ[3] game

x	y	z	$a \oplus b \oplus c$
0	0	0	$0 \mod 2$
0	1	1	$1 \mod 2$
1	0	1	$1 \mod 2$
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[3] Greenberger, Horne, and Zeilinger 1989

GHZ[3] game

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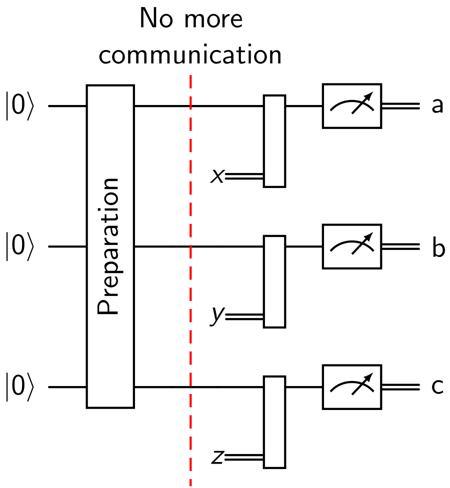
1	0	1	1 mod 2
1	1	0	1 mod 2



[3] Greenberger, Horne, and Zeilinger 1989

Distributed Quantum Advantage for Local Problems

Summary

	Classic	Quantum	Local
Previous work[4]	$\Omega(n)$	$O(1)$	No
Our work	$\Theta(\Delta)$	$O(1)$	Yes

[4] Le Gall, Nishimura, and Rosmanis 2019

Summary

	Classic	Quantum	Local
Previous work[4]	$\Omega(n)$	$O(1)$	No
Our work	$\Theta(\Delta)$	$O(1)$	Yes
Future work	$\Omega(\log n \cdot \frac{\log\log n}{\log\log\log n})$	$O(\log n)$	Yes

[4] Le Gall, Nishimura, and Rosmanis 2019

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